



# Fundamentals of Music Processing

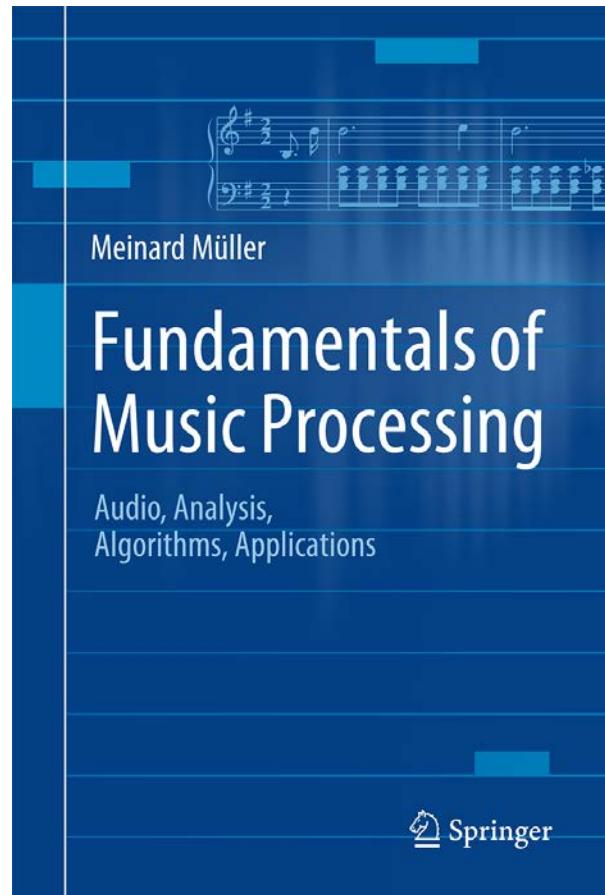
## Chapter 8: Musically Informed Audio Decomposition

Meinard Müller

International Audio Laboratories Erlangen

[www.music-processing.de](http://www.music-processing.de)

# Book: Fundamentals of Music Processing

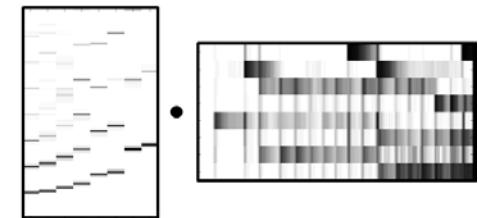


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# Chapter 8: Musically Informed Audio Decomposition

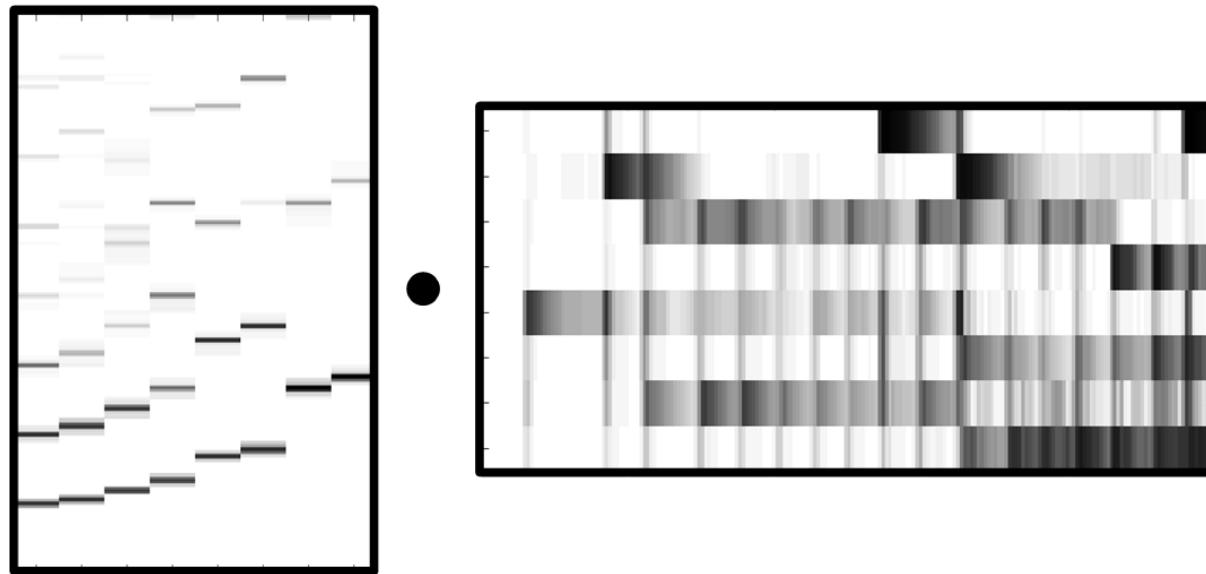
- 8.1 Harmonic-Percussive Separation
- 8.2 Melody Extraction
- 8.3 NMF-Based Audio Decomposition
- 8.4 Further Notes



In the final Chapter 8 on audio decomposition, we present a challenging research direction that is closely related to source separation. Within this wide research area, we consider three subproblems: harmonic–percussive separation, main melody extraction, and score-informed audio decomposition. Within these scenarios, we discuss a number of key techniques including instantaneous frequency estimation, fundamental frequency ( $F_0$ ) estimation, spectrogram inversion, and nonnegative matrix factorization (NMF). Furthermore, we encounter a number of acoustic and musical properties of audio recordings that have been introduced and discussed in previous chapters, which rounds off the book.

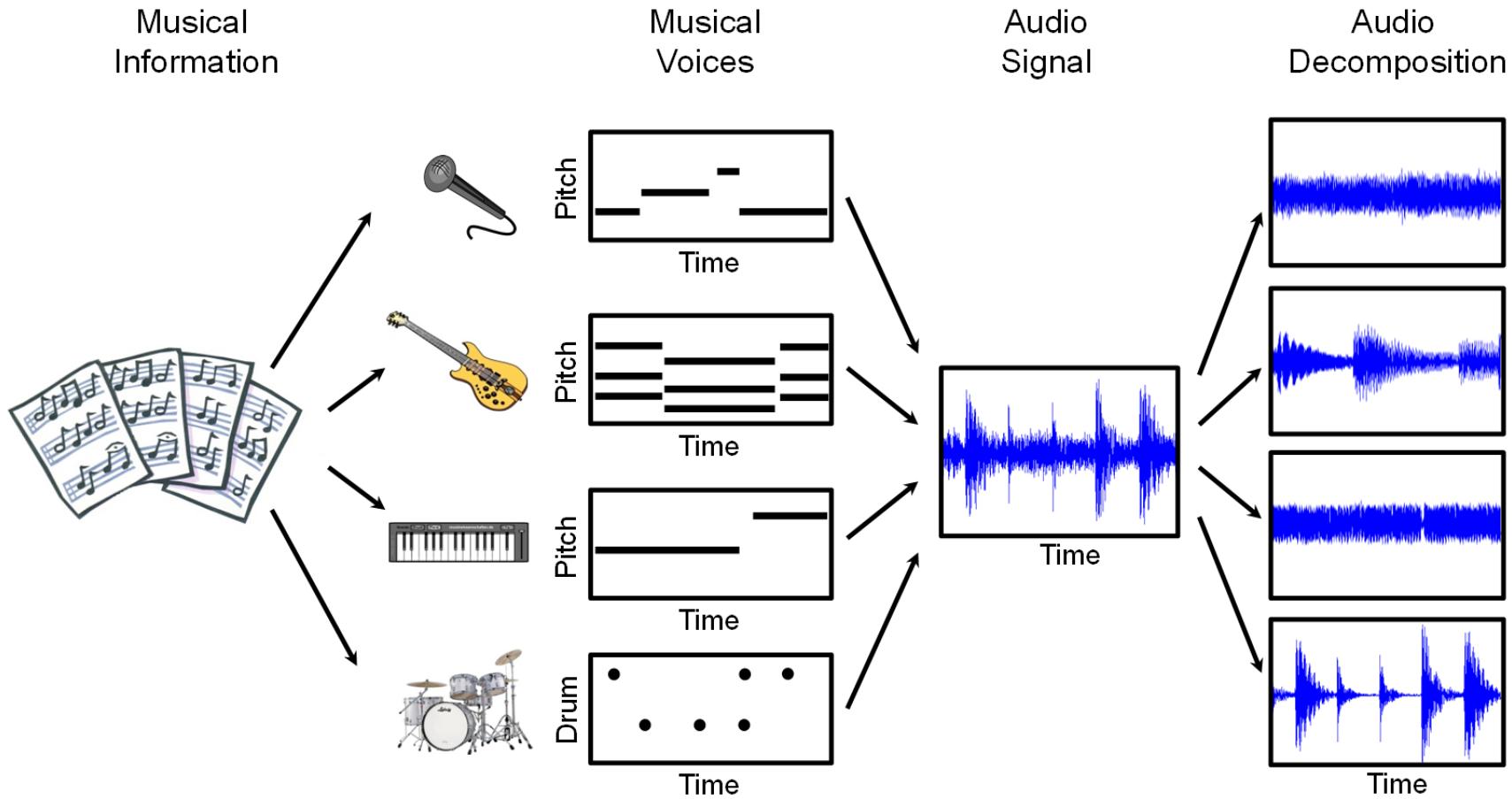
# 8 Content-Based Audio Retrieval

Teaser



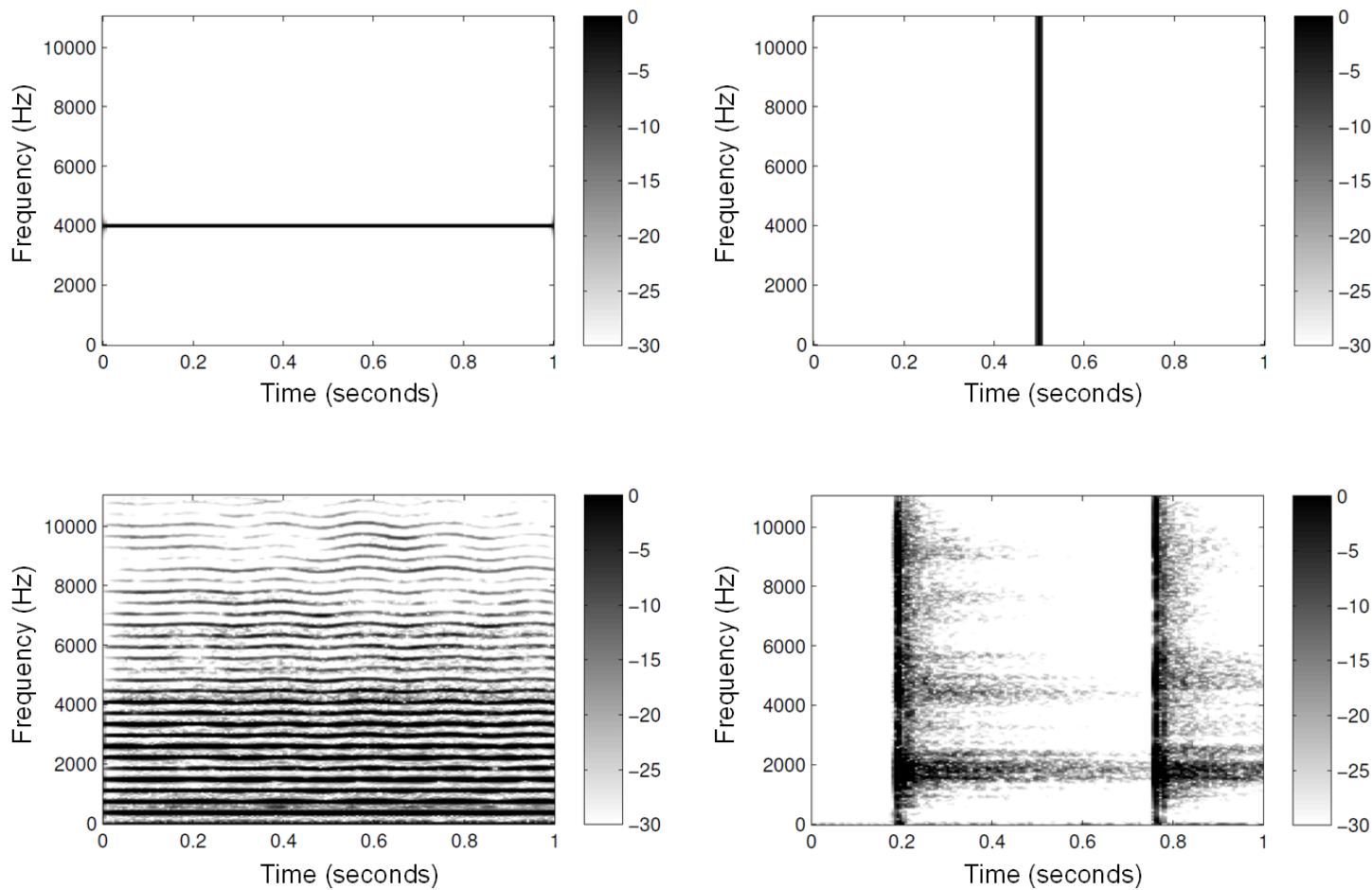
# 8 Content-Based Audio Retrieval

Fig. 8.1



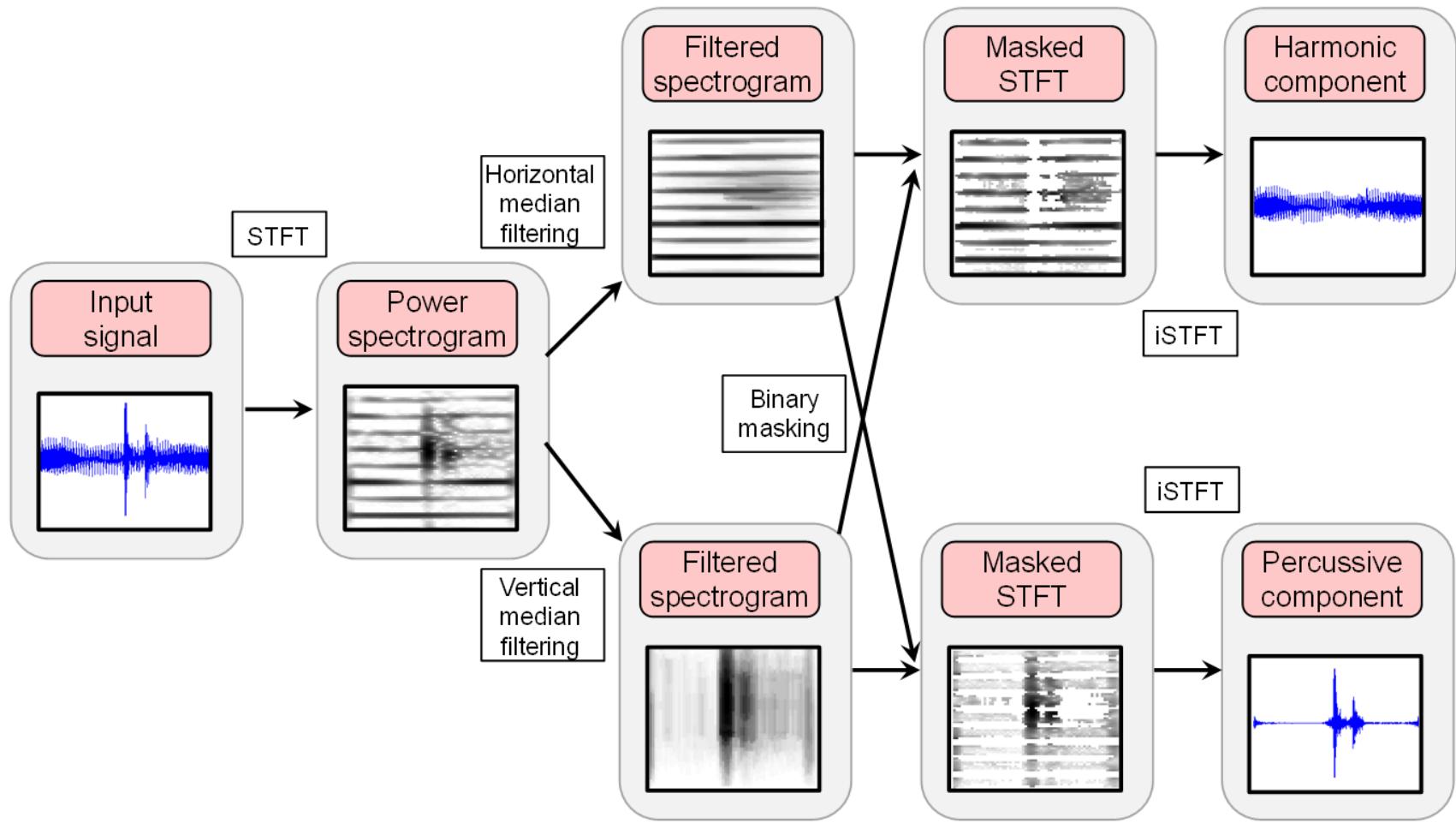
# 8.1 Harmonic–Percussive Separation

Fig. 8.2



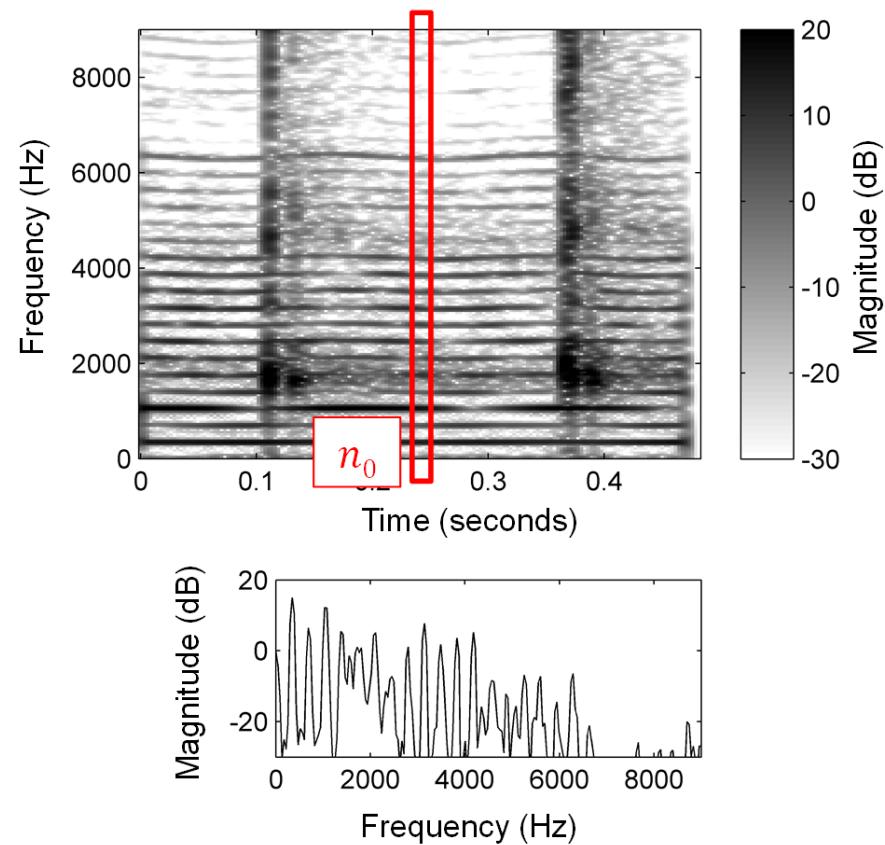
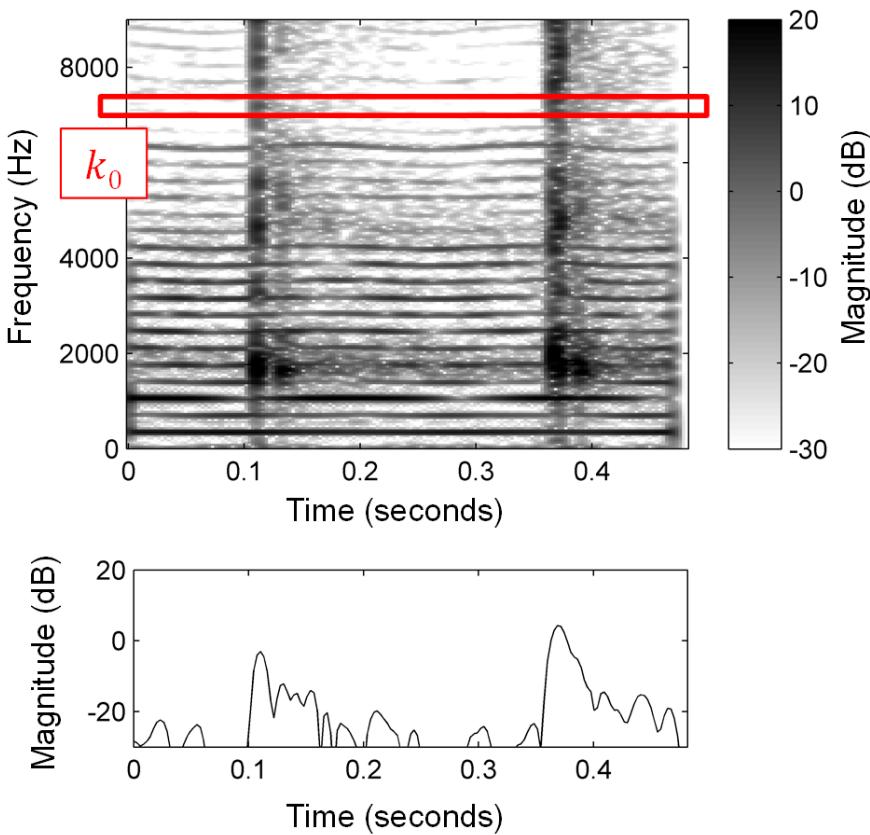
# 8.1 Harmonic–Percussive Separation

Fig. 8.3



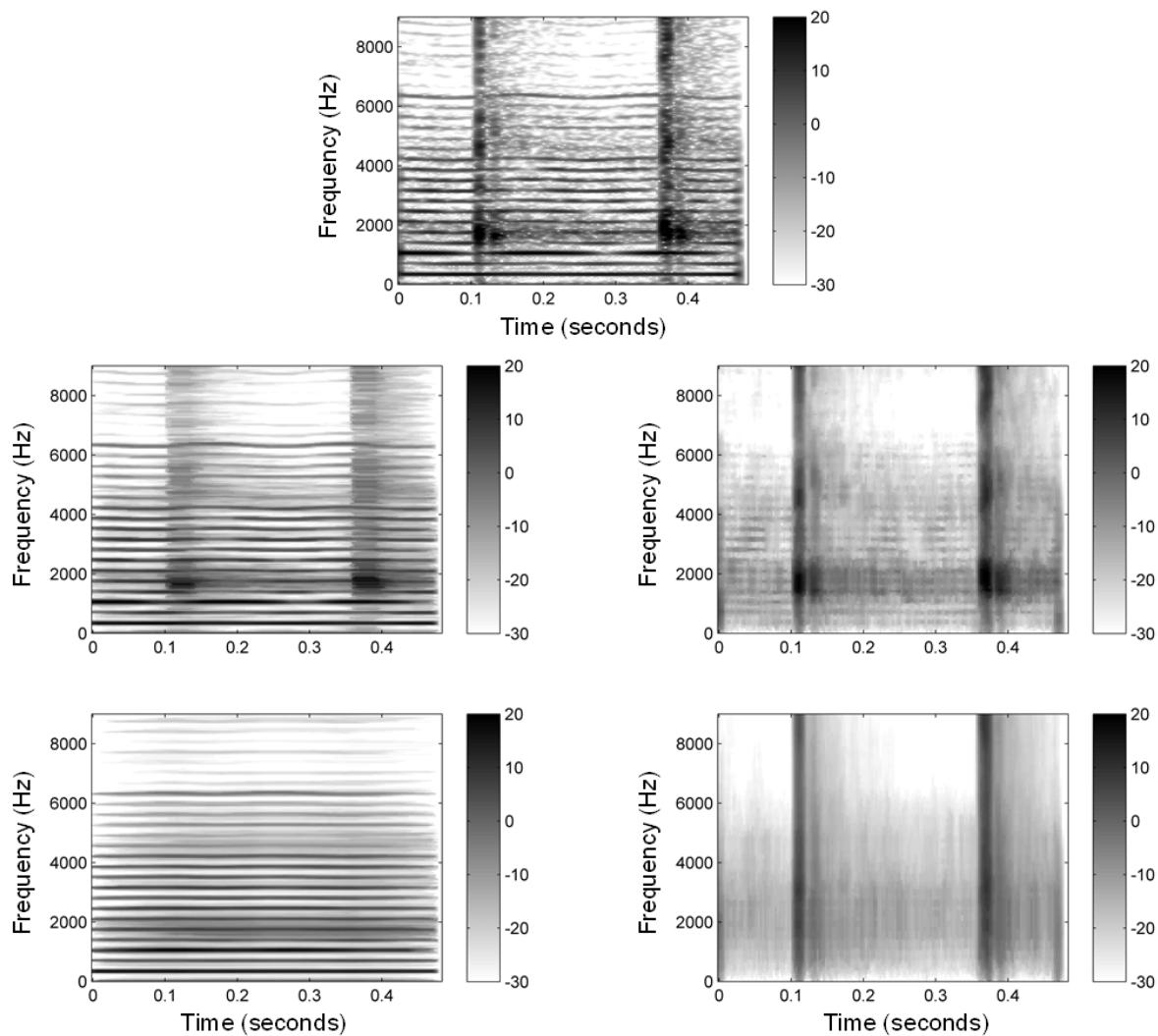
# 8.1 Harmonic–Percussive Separation

Fig. 8.4



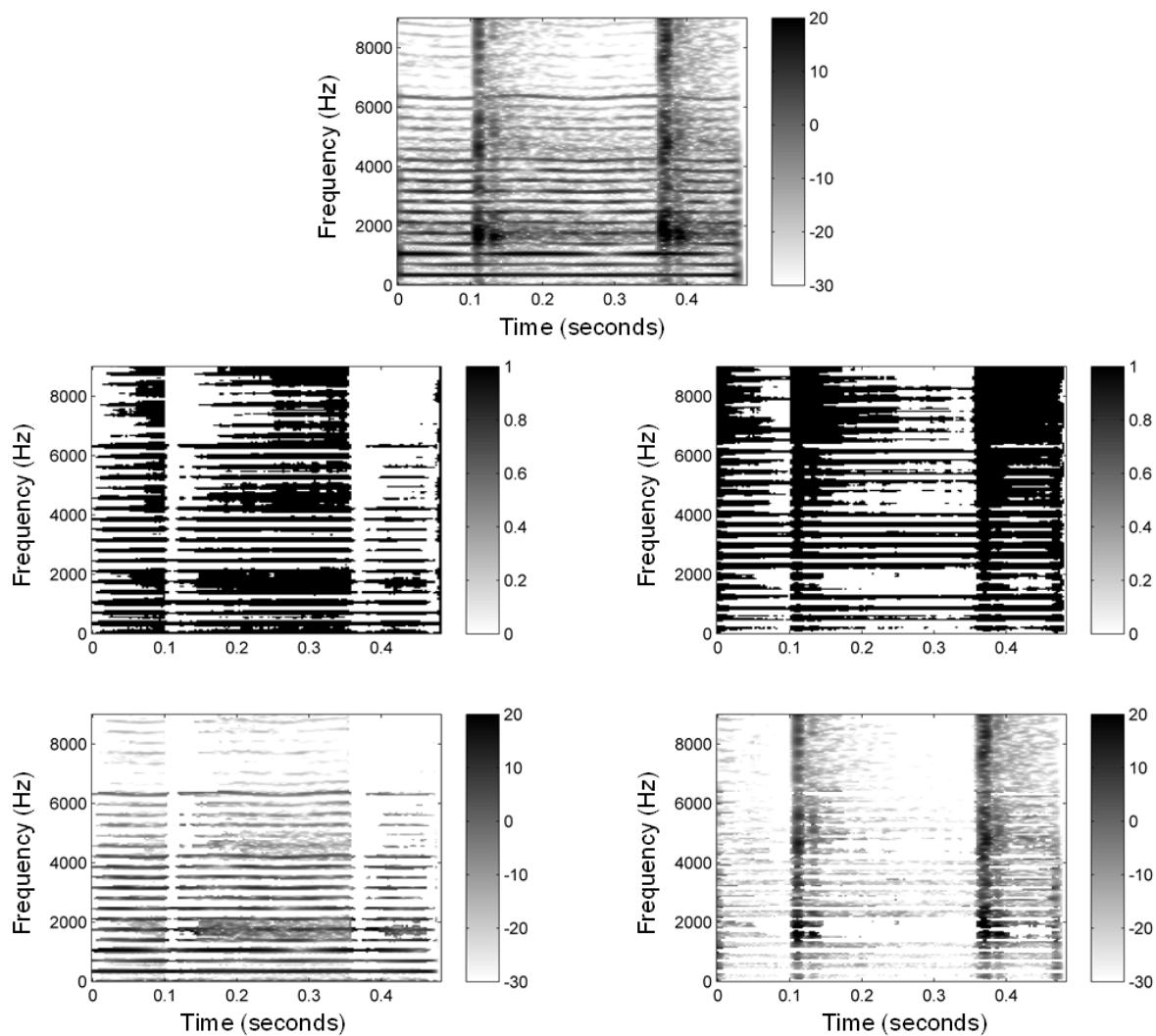
# 8.1 Harmonic–Percussive Separation

Fig. 8.5



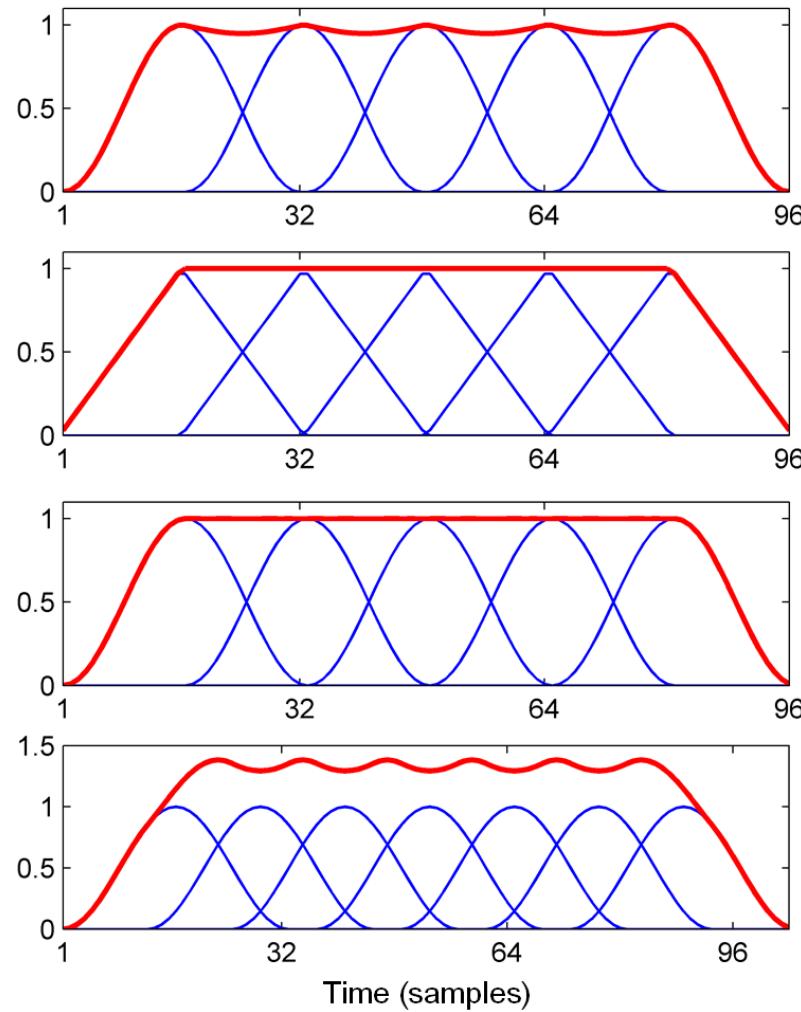
# 8.1 Harmonic–Percussive Separation

Fig. 8.6



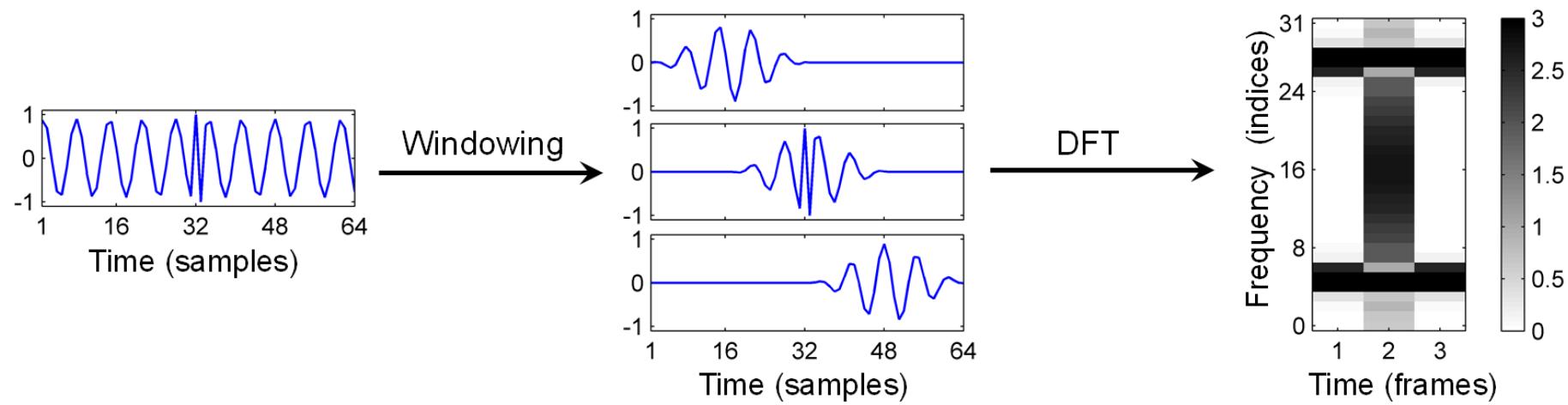
# 8.1 Harmonic–Percussive Separation

Fig. 8.7



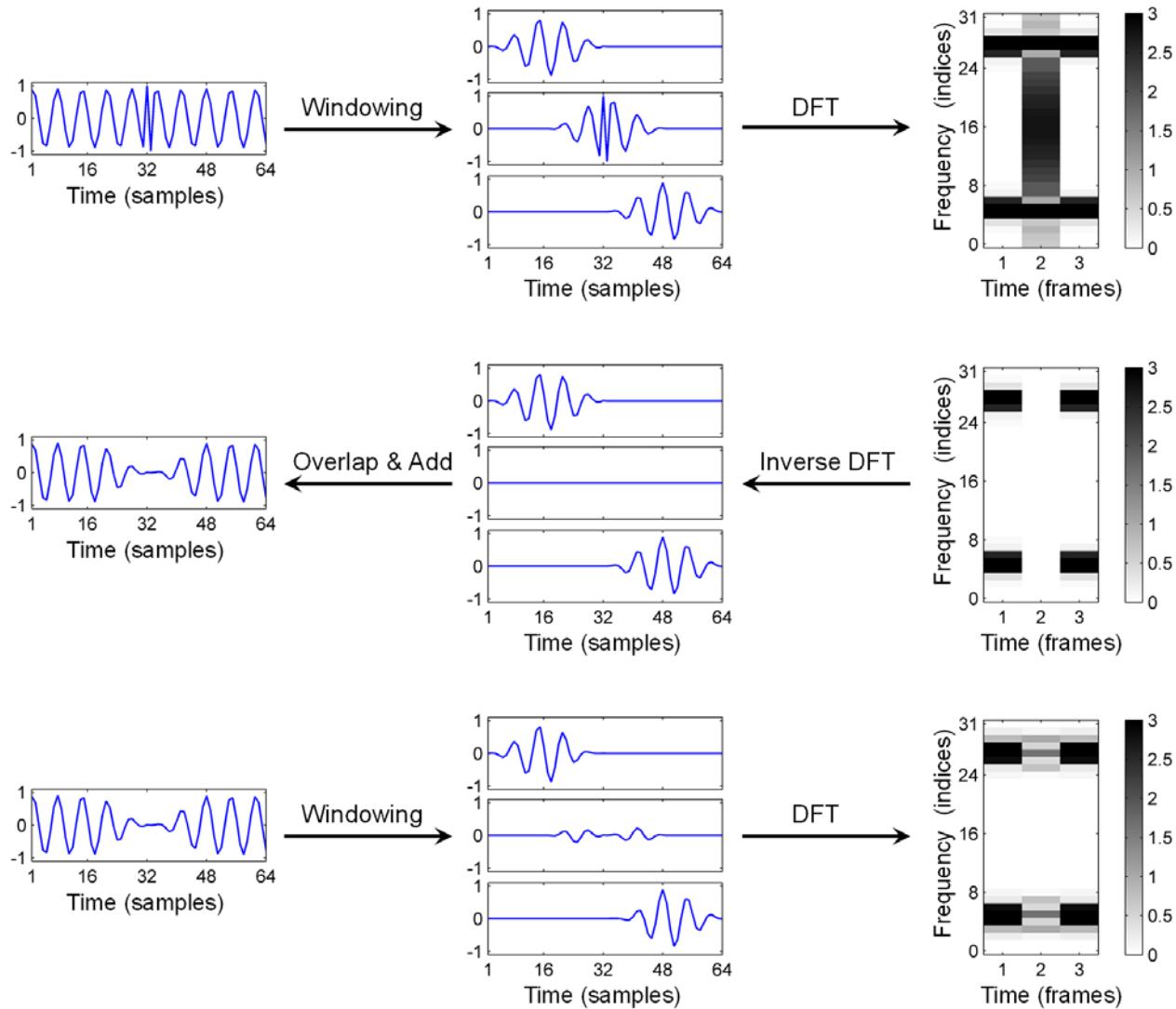
# 8.1 Harmonic–Percussive Separation

Fig. 8.8



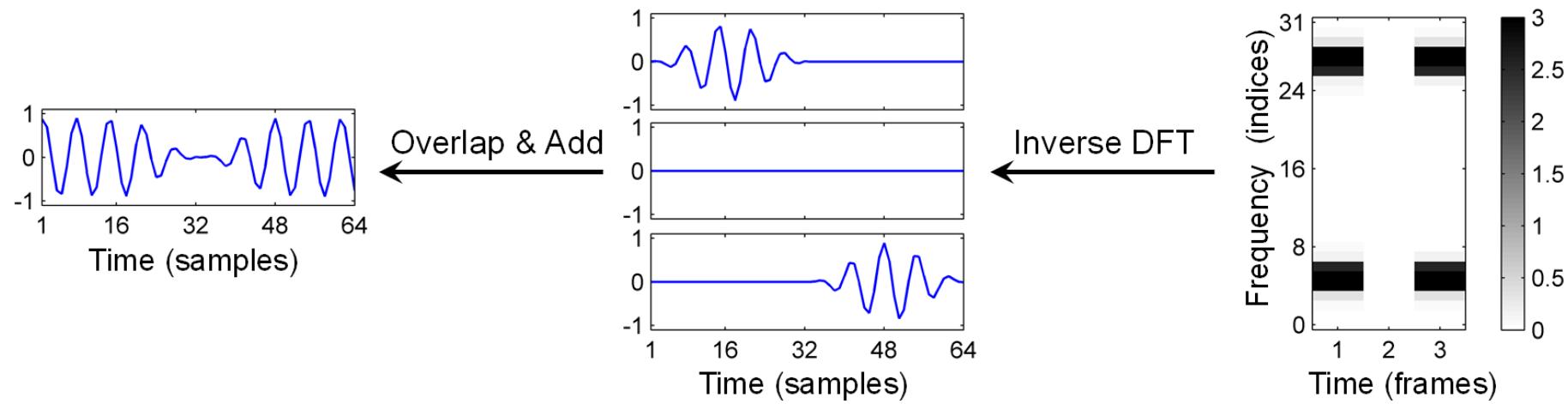
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Fig. 8.8



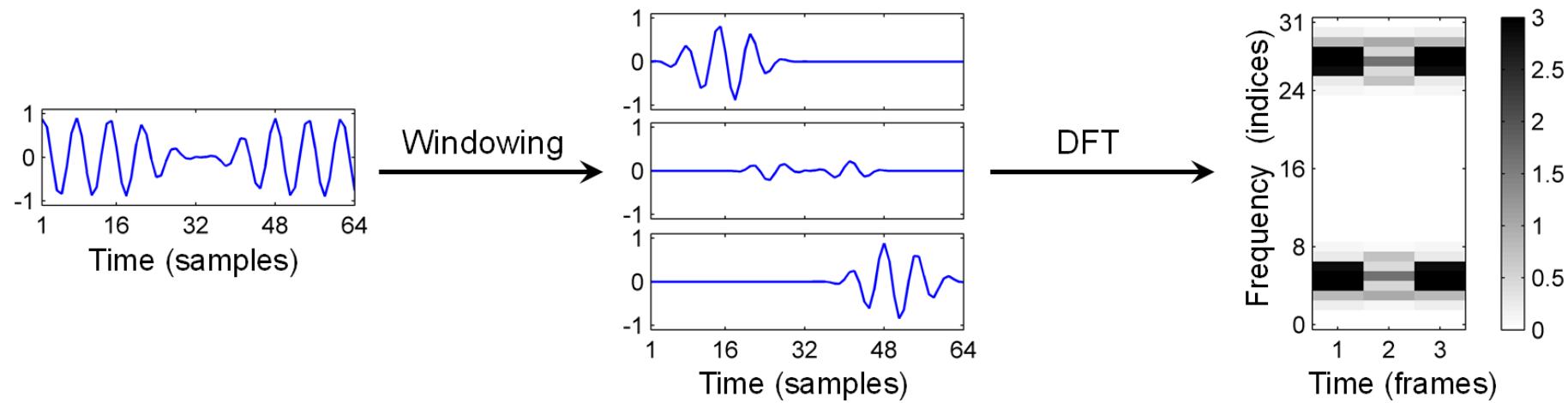
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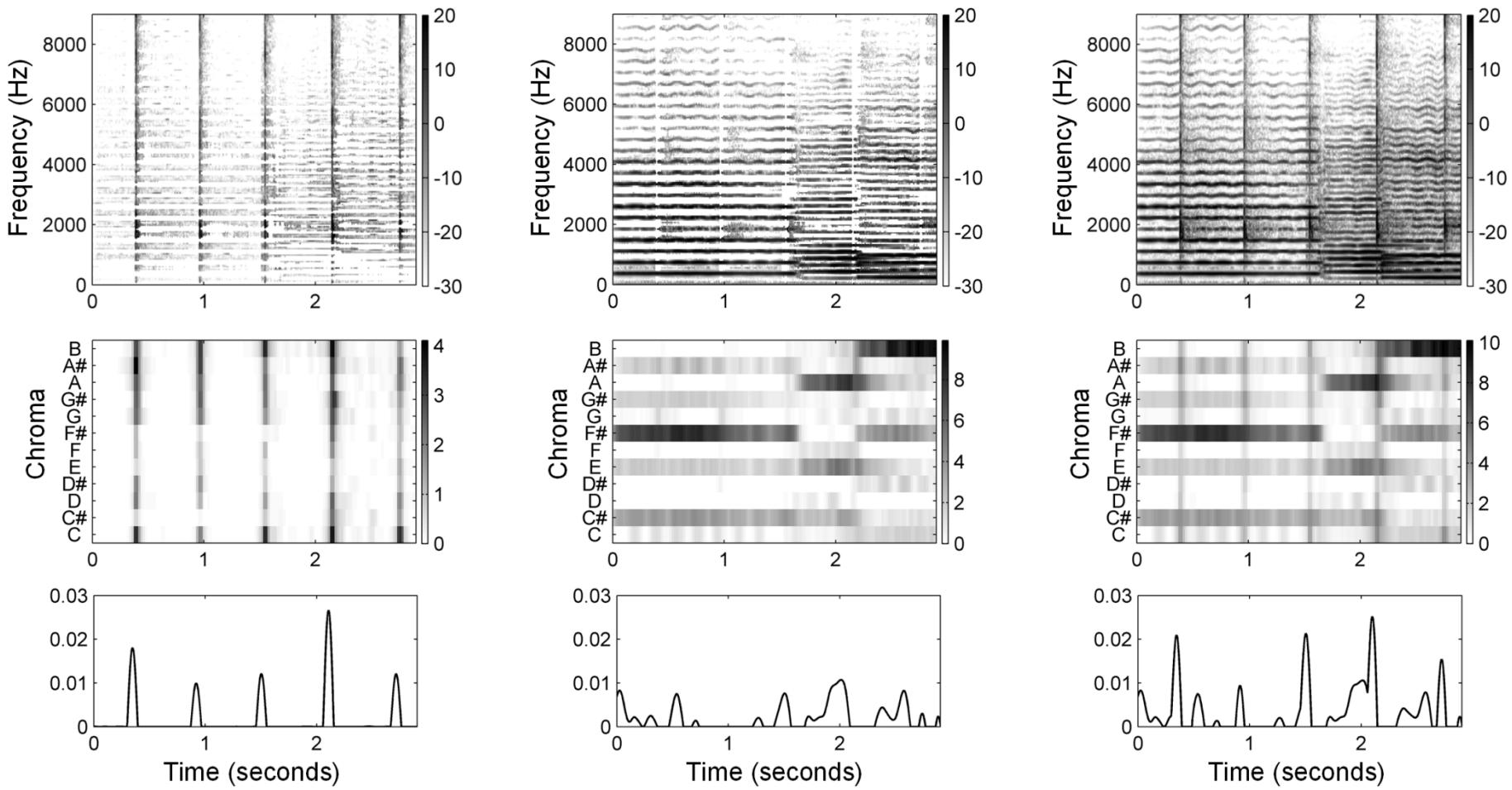
# 8.1 Harmonic–Percussive Separation

Fig. 8.8



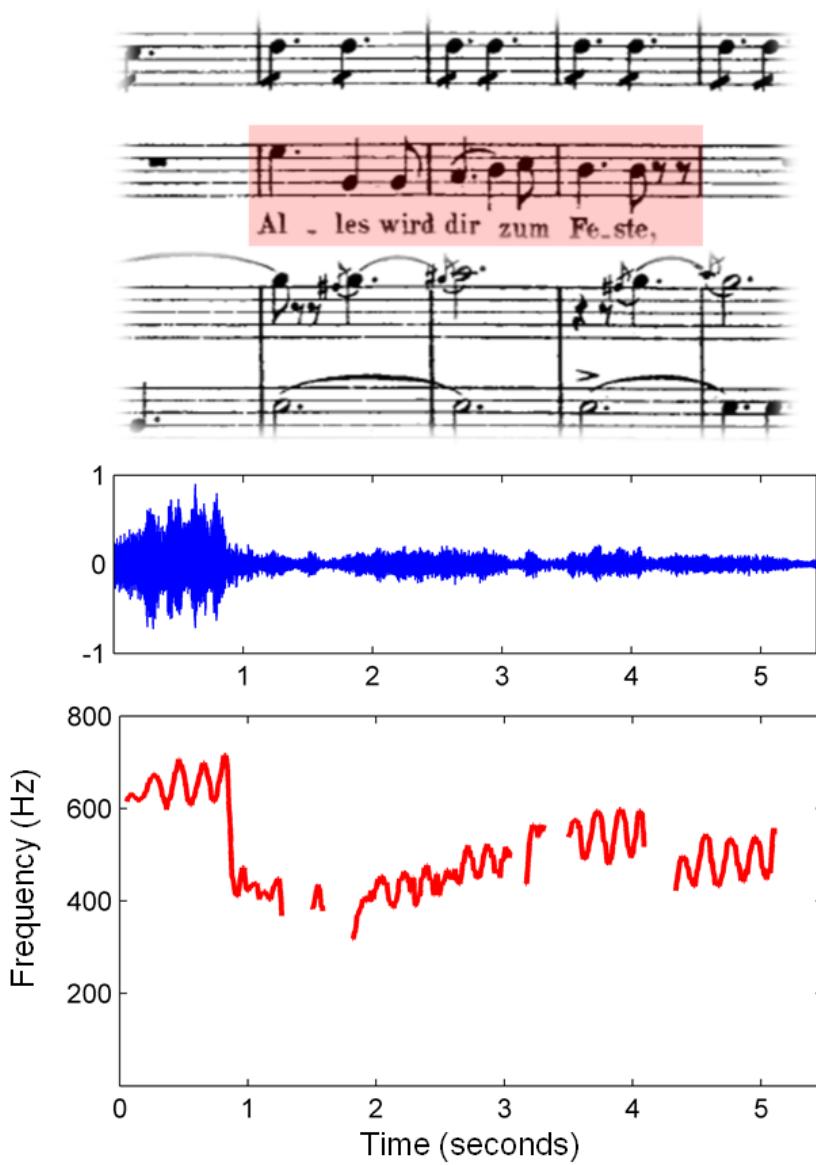
# 8.1 Harmonic–Percussive Separation

Fig. 8.9



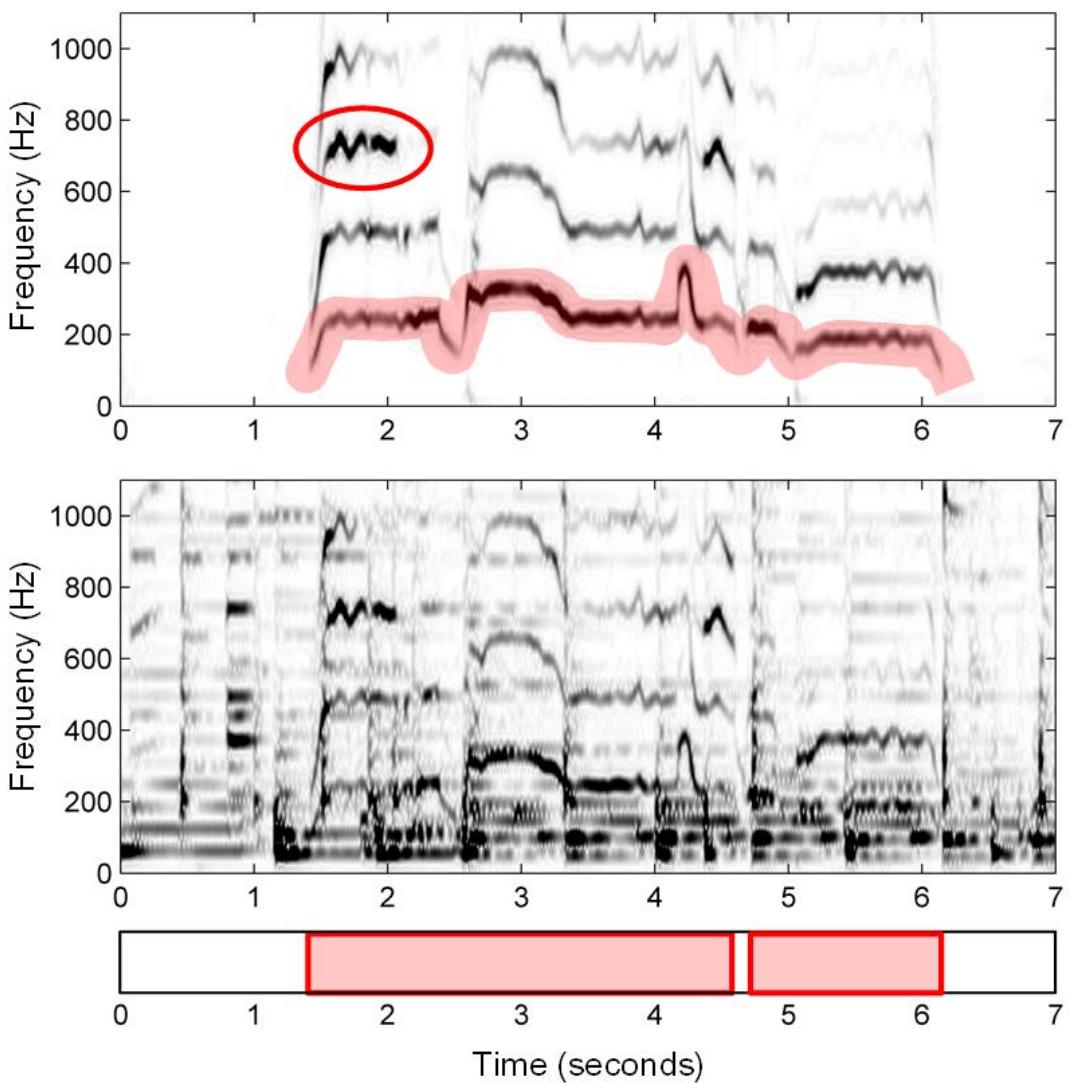
## 8.2 Melody Extraction

Fig. 8.10



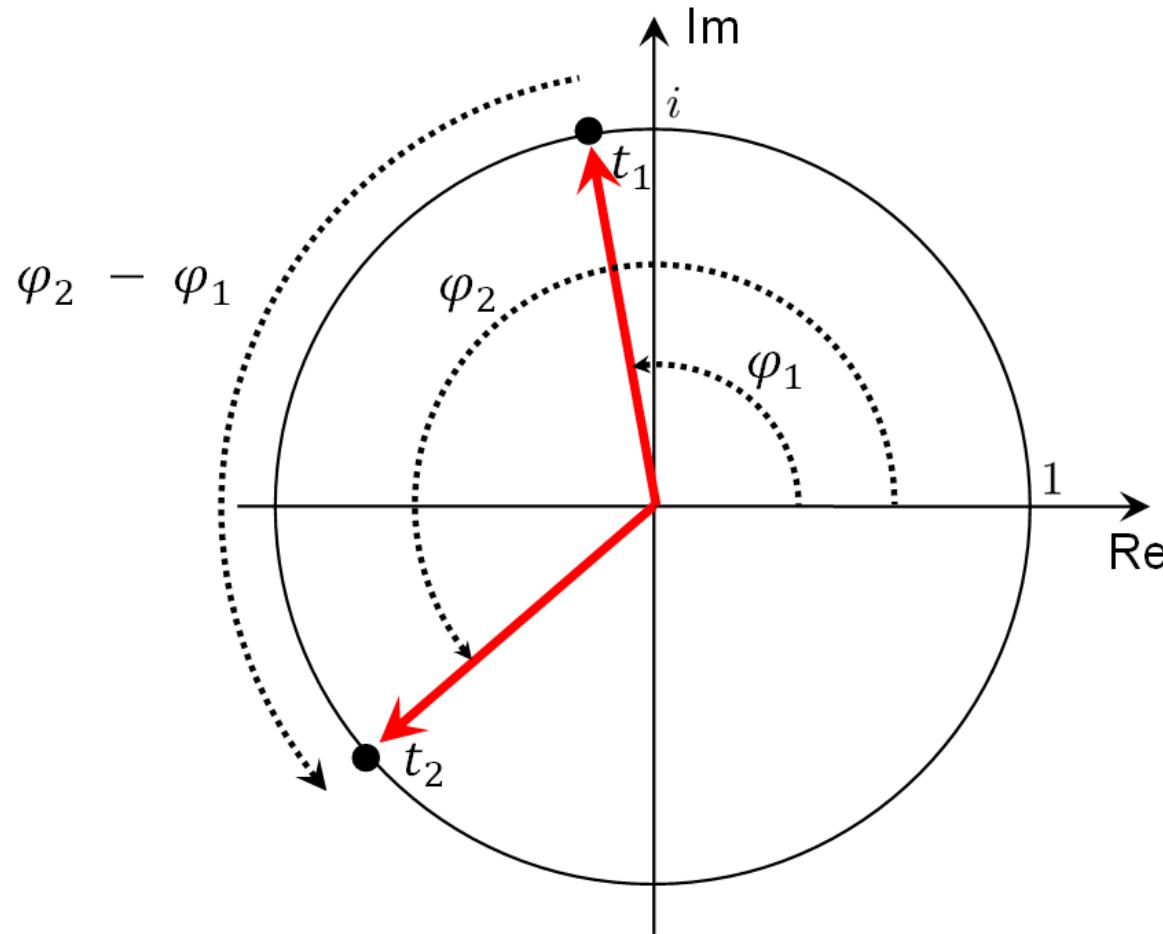
## 8.2 Melody Extraction

Fig. 8.11



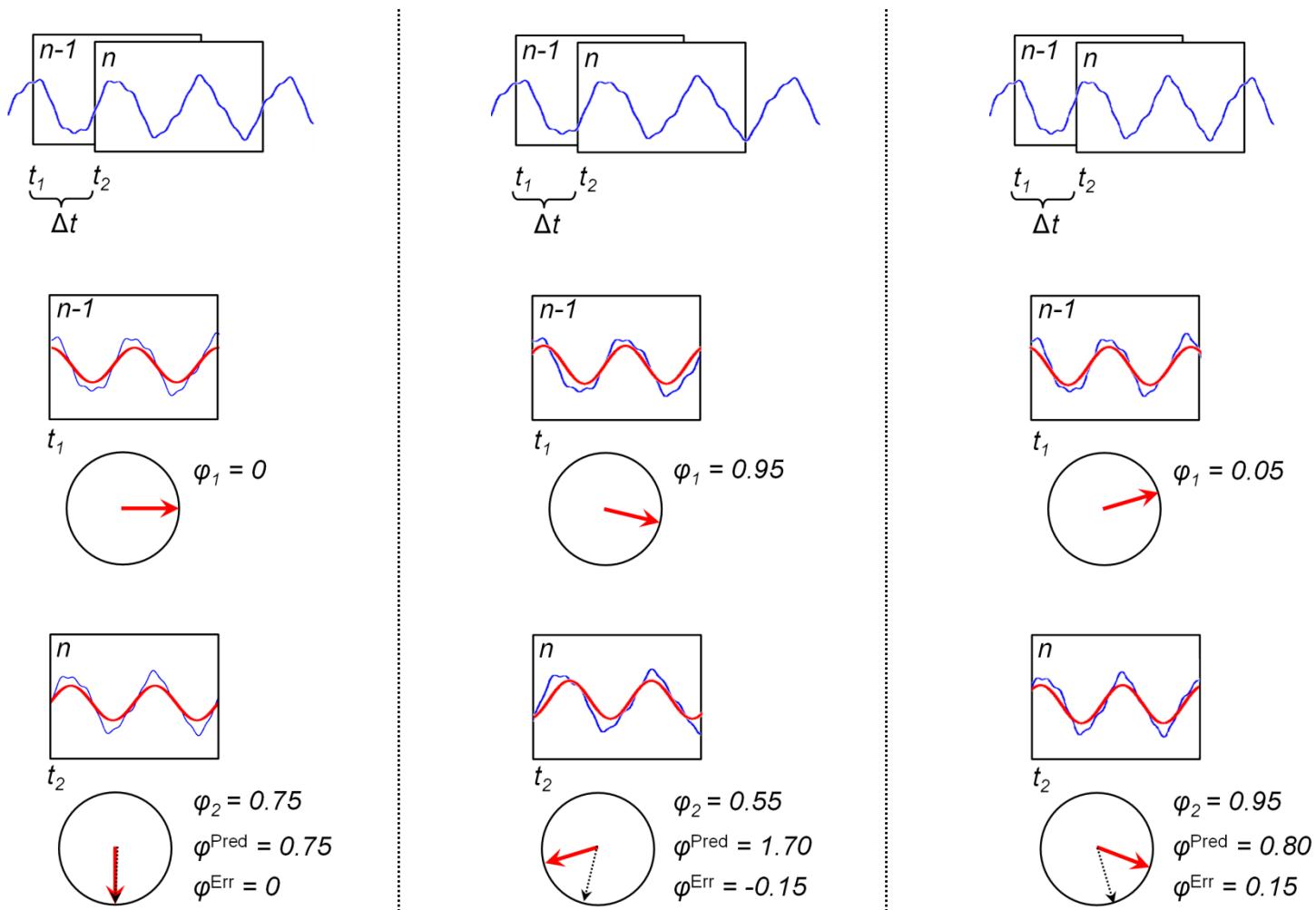
## 8.2 Melody Extraction

Fig. 8.12



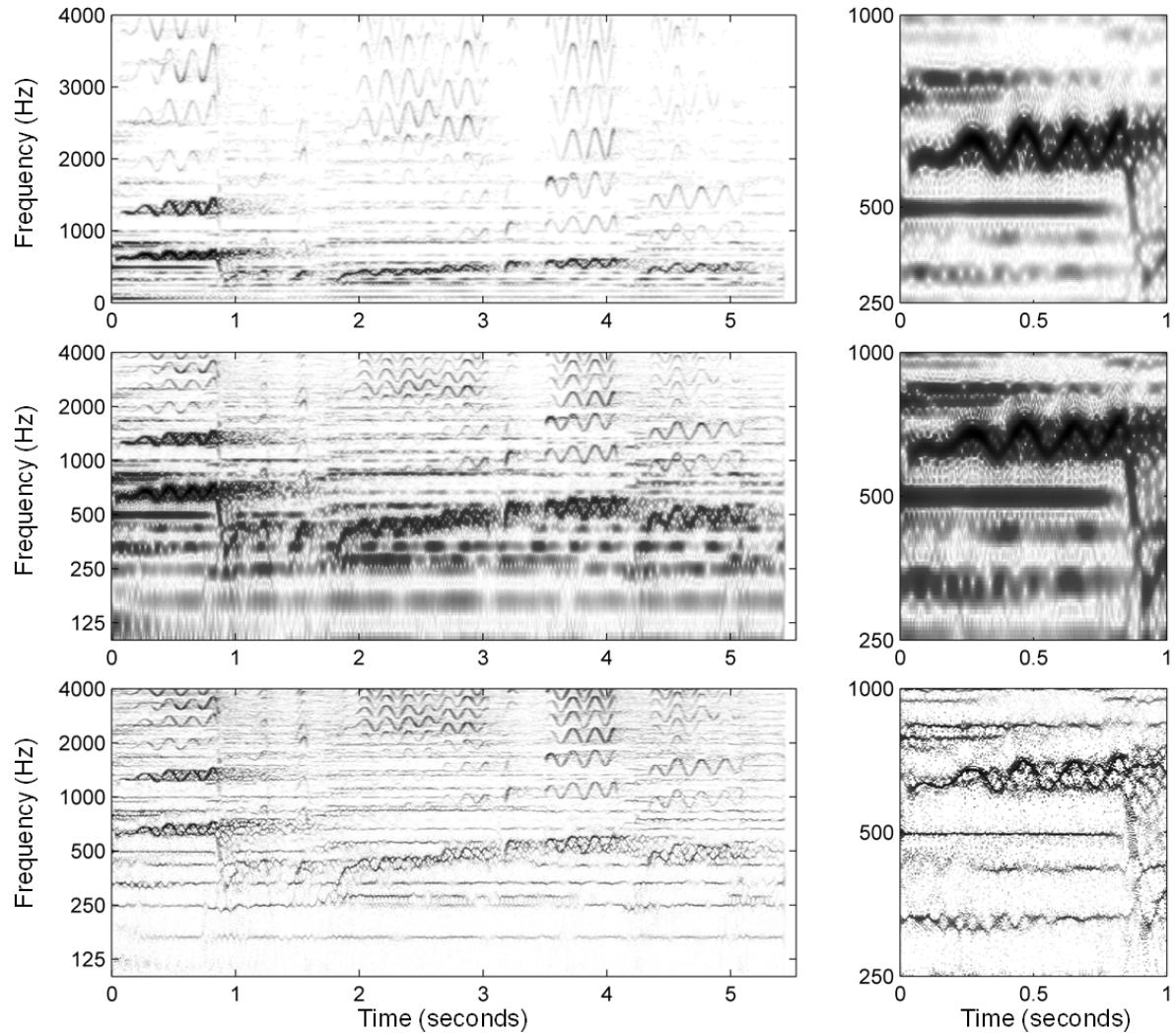
## 8.2 Melody Extraction

Fig. 8.13



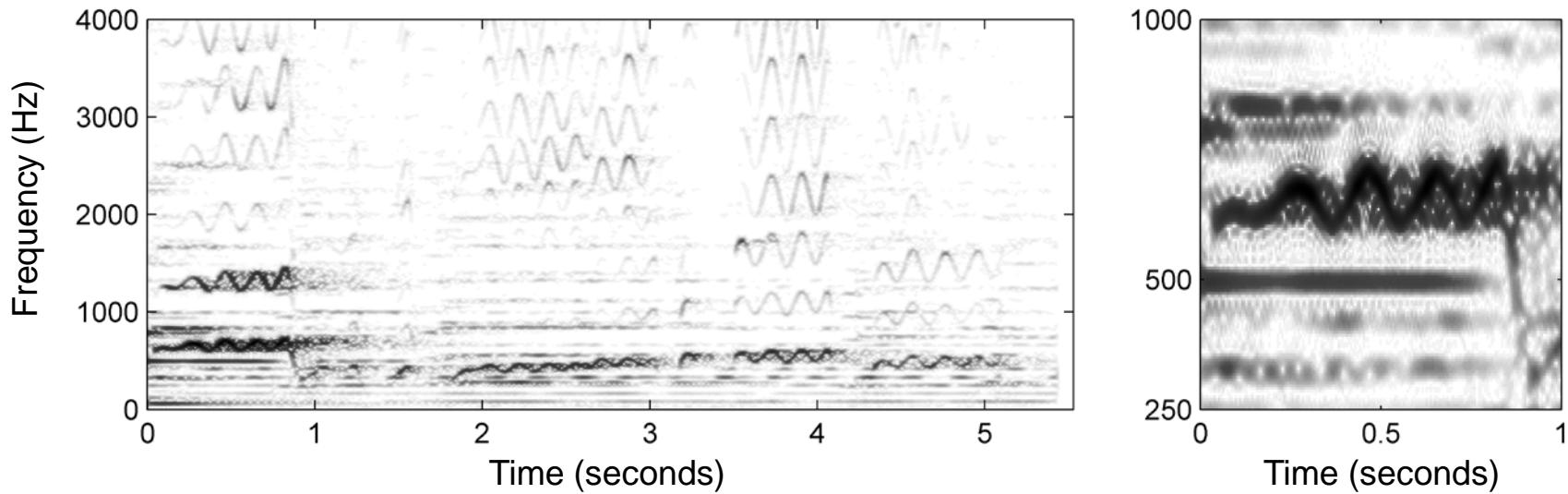
## 8.2 Melody Extraction

Fig. 8.14



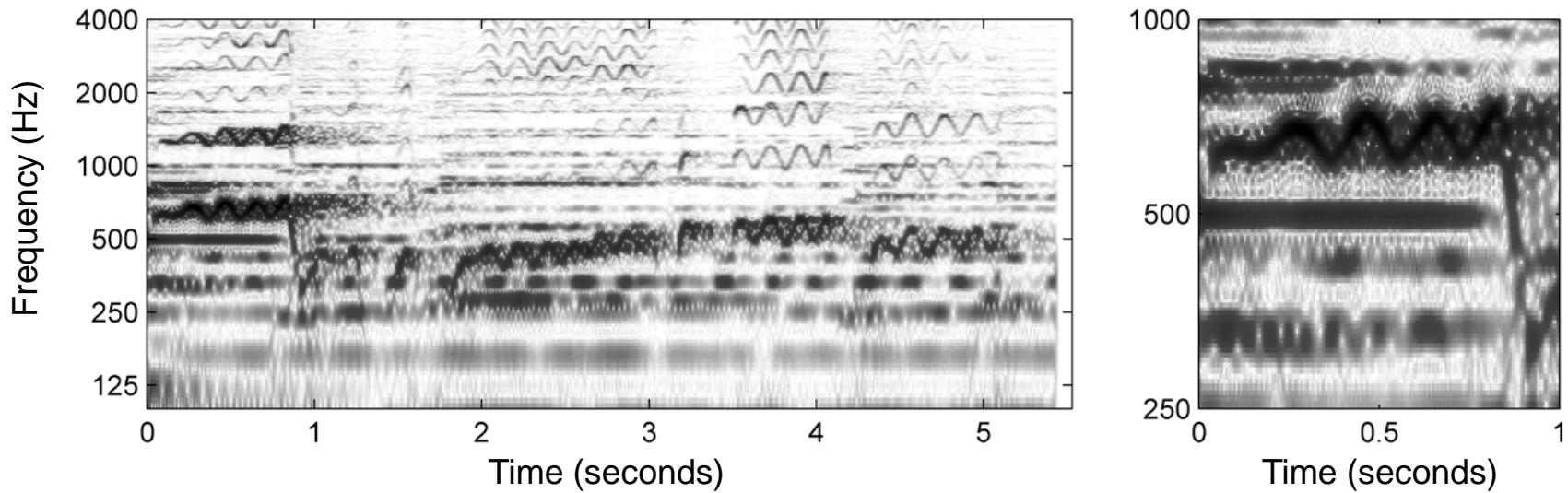
## 8.2 Melody Extraction

Fig. 8.14



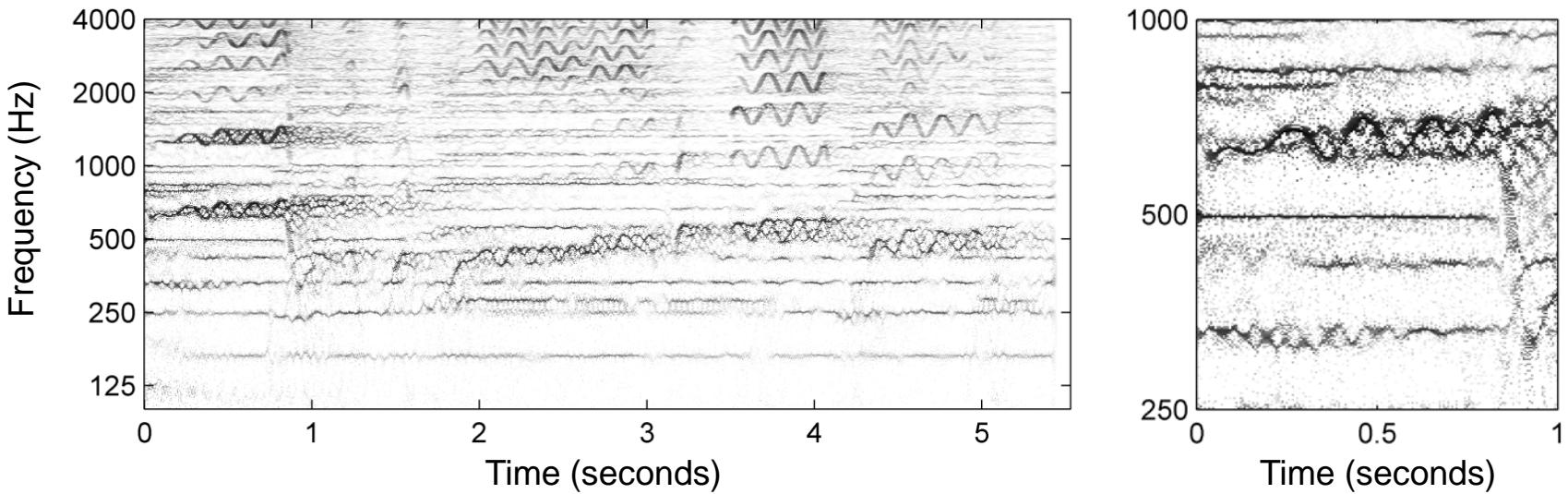
## 8.2 Melody Extraction

Fig. 8.14



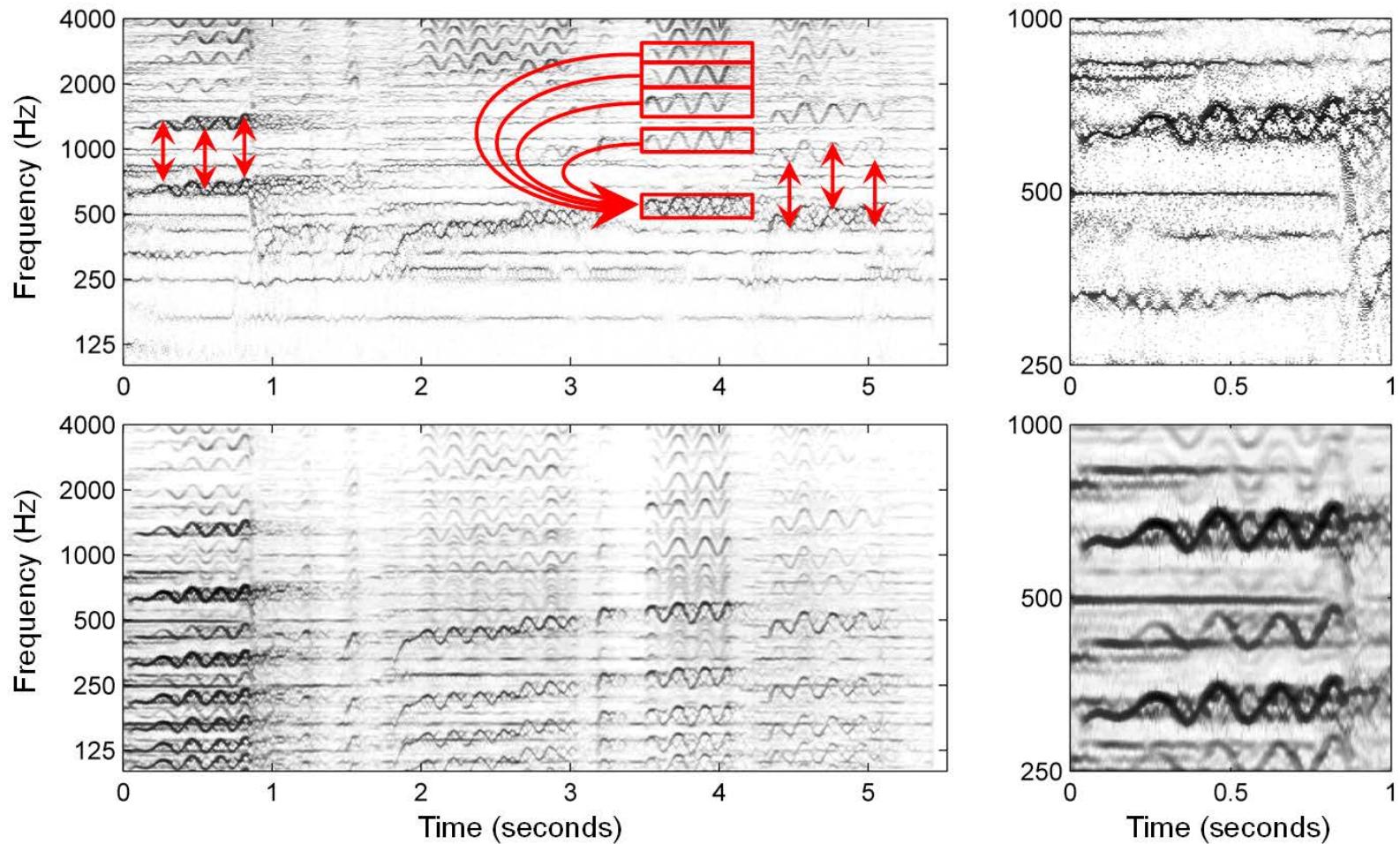
## 8.2 Melody Extraction

Fig. 8.14



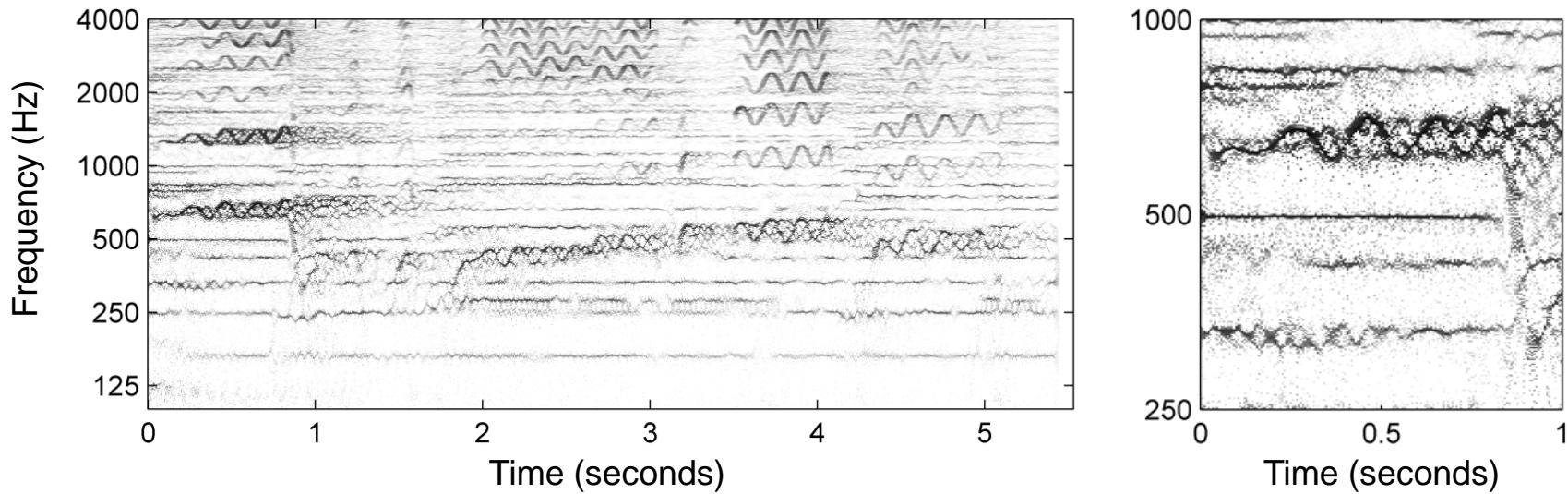
## 8.2 Melody Extraction

Fig. 8.15



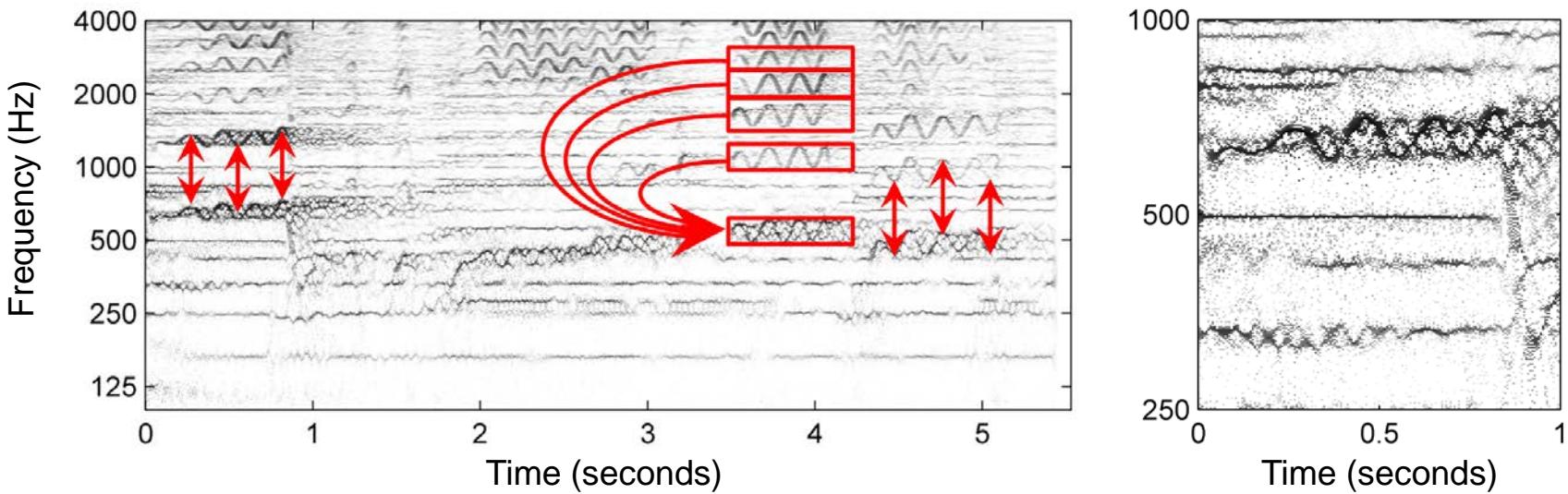
## 8.2 Melody Extraction

Fig. 8.15



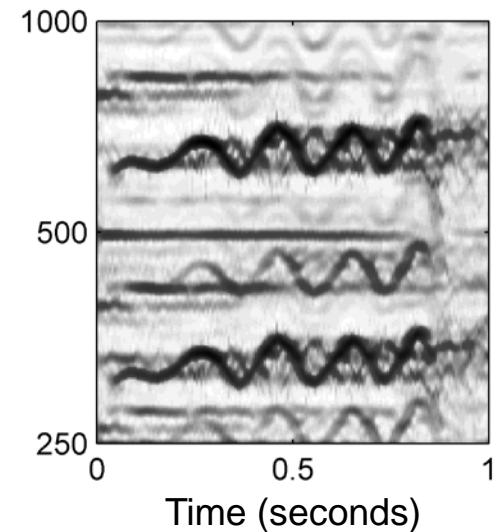
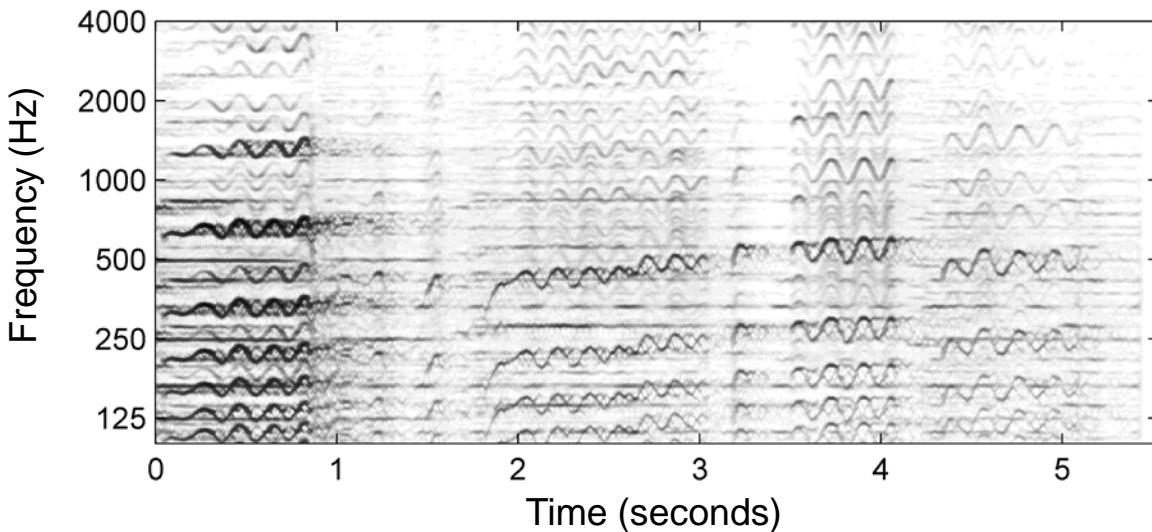
## 8.2 Melody Extraction

Fig. 8.15



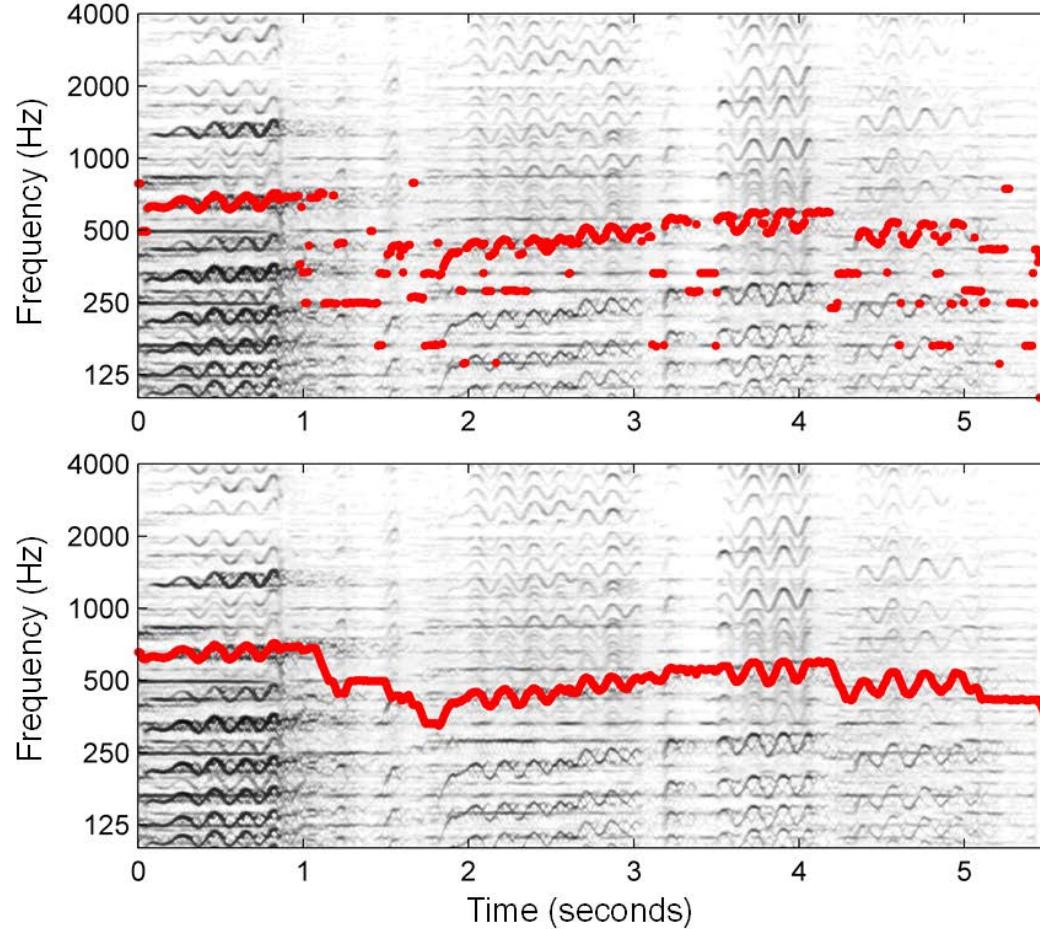
## 8.2 Melody Extraction

Fig. 8.15



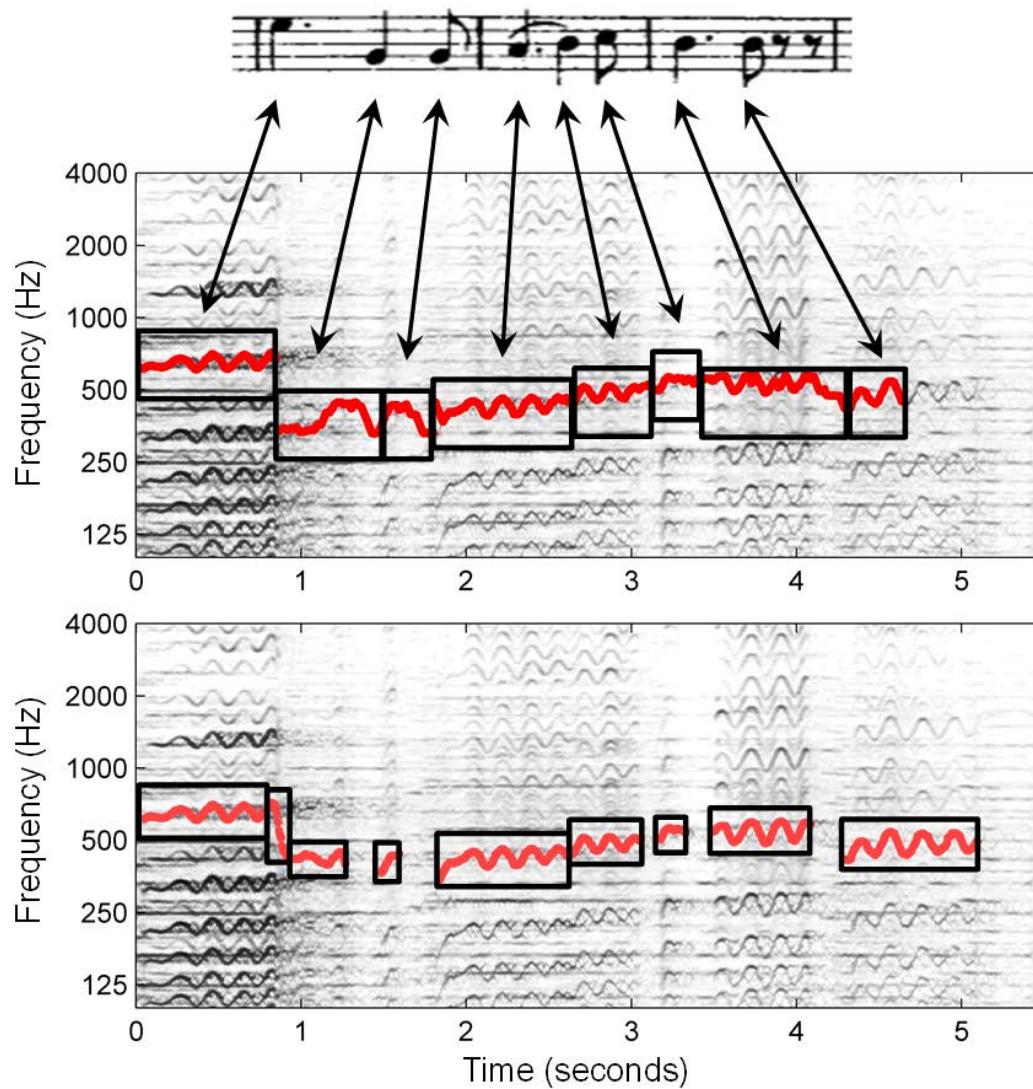
## 8.2 Melody Extraction

Fig. 8.16



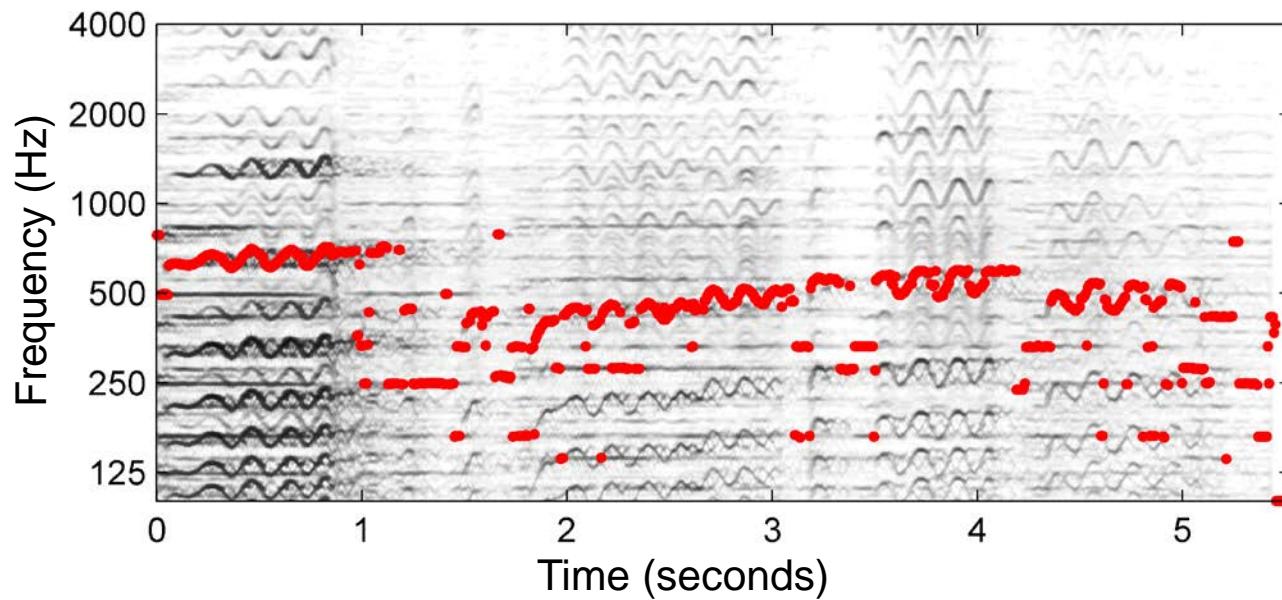
## 8.2 Melody Extraction

Fig. 8.17



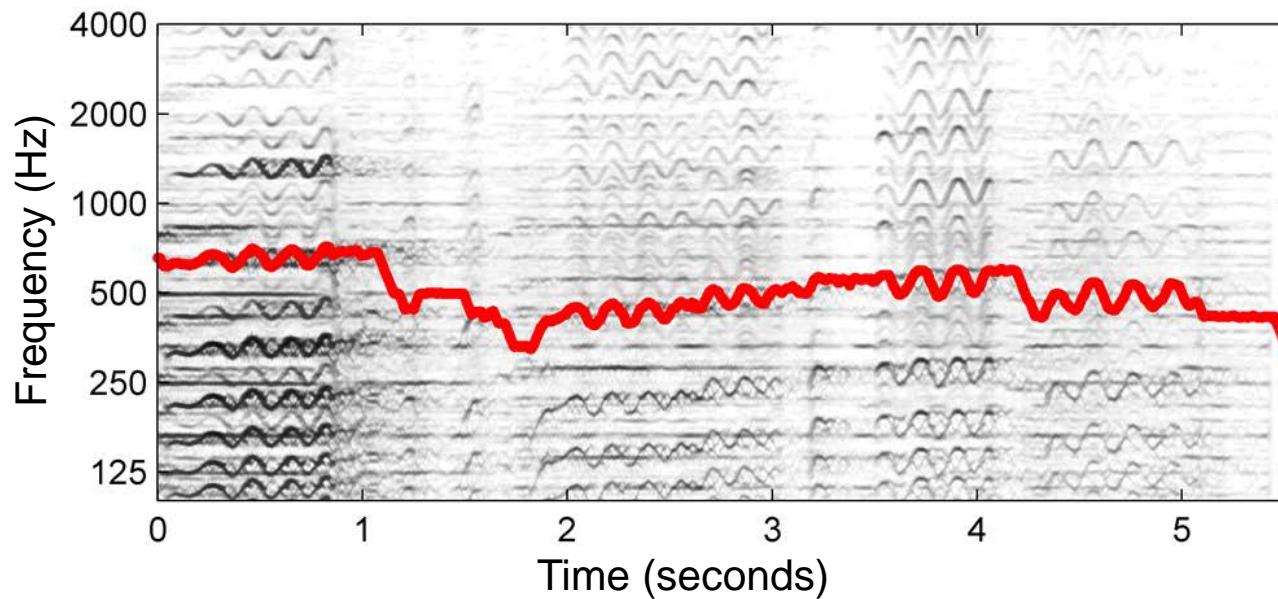
## 8.2 Melody Extraction

Fig. 8.16



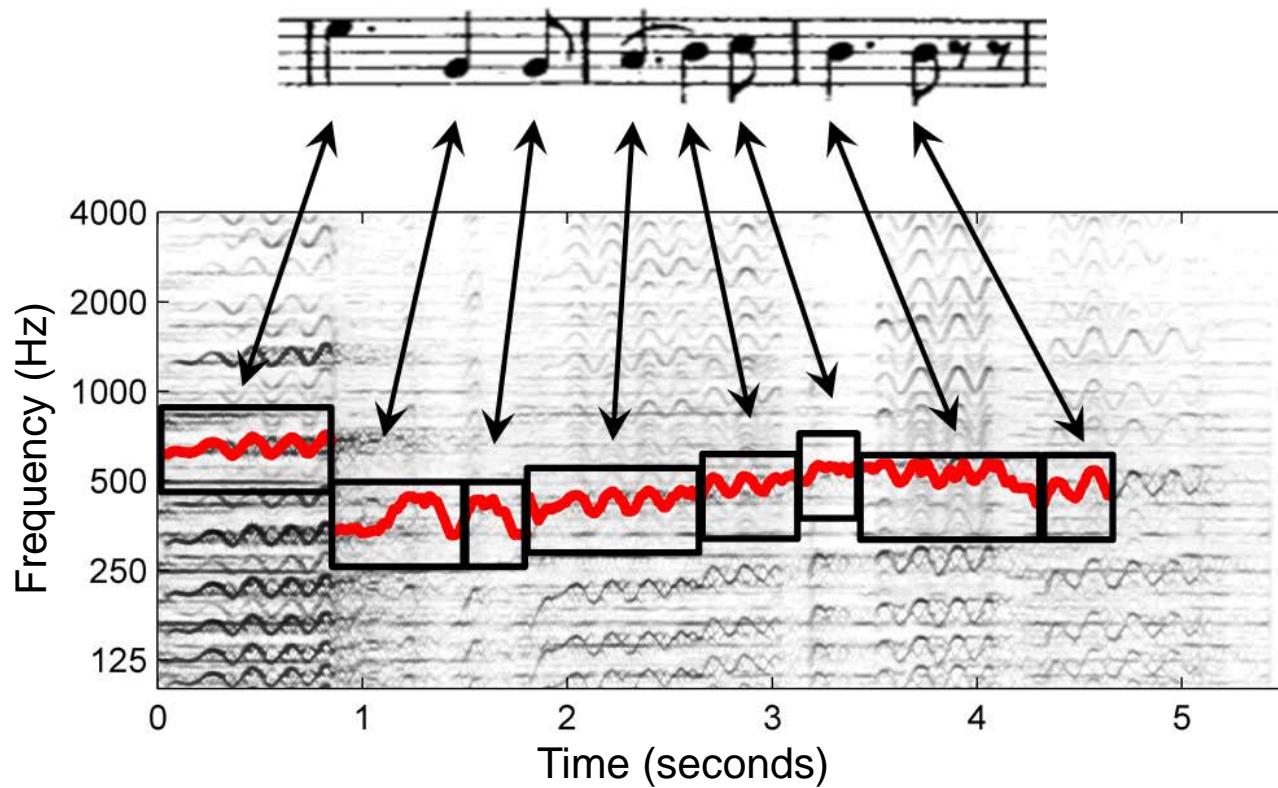
## 8.2 Melody Extraction

Fig. 8.16



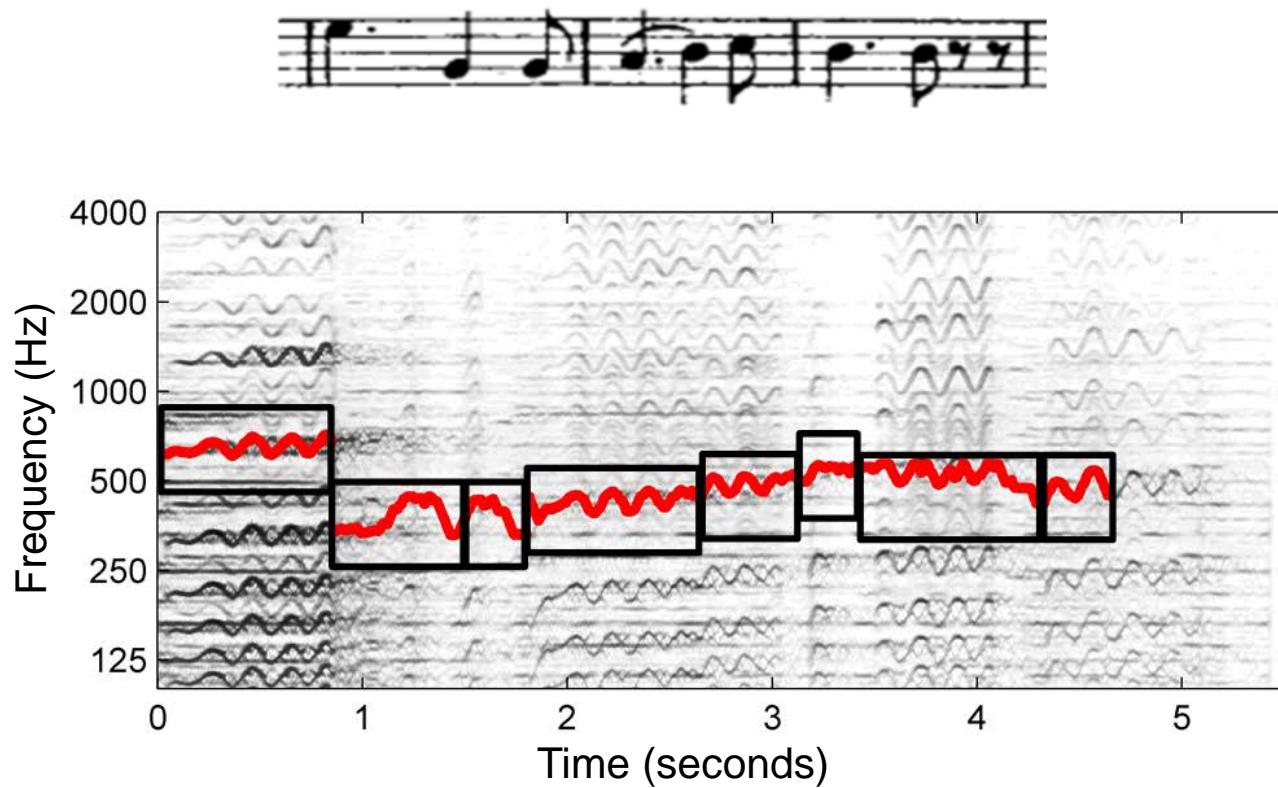
## 8.2 Melody Extraction

Fig. 8.17



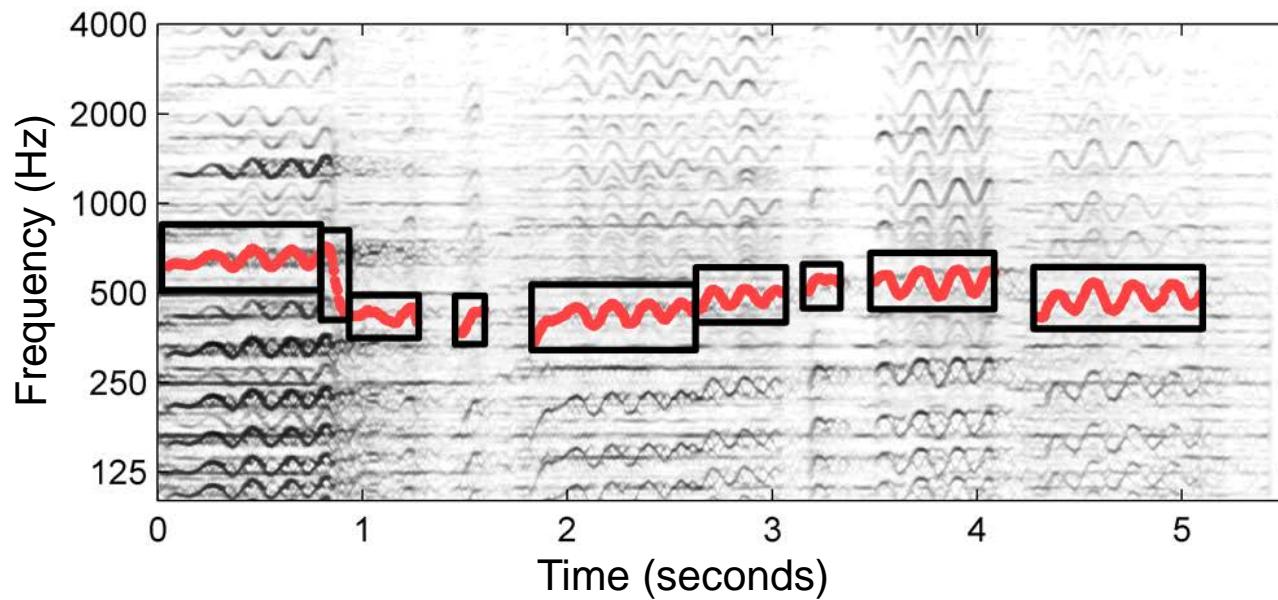
## 8.2 Melody Extraction

Fig. 8.17



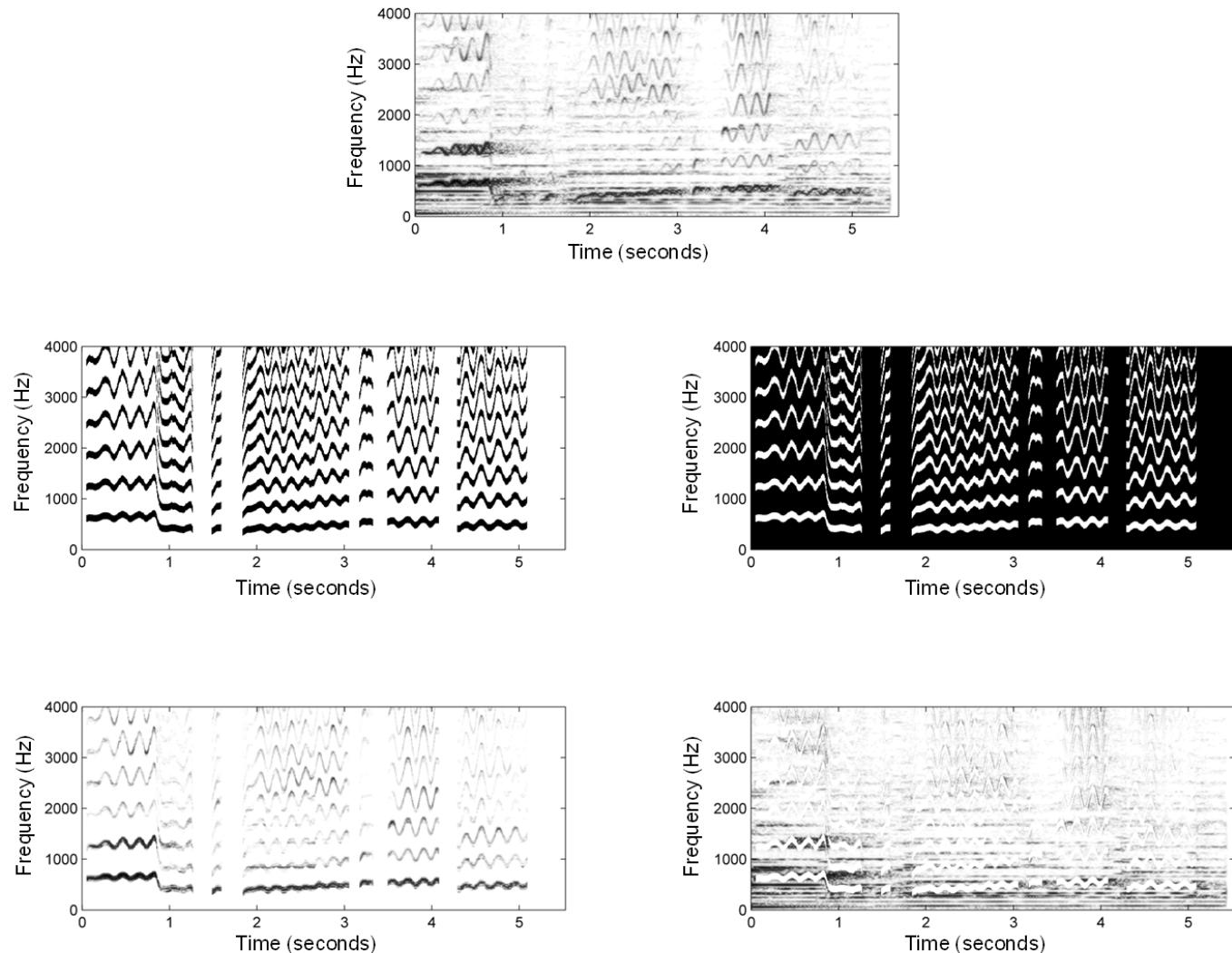
## 8.2 Melody Extraction

Fig. 8.17



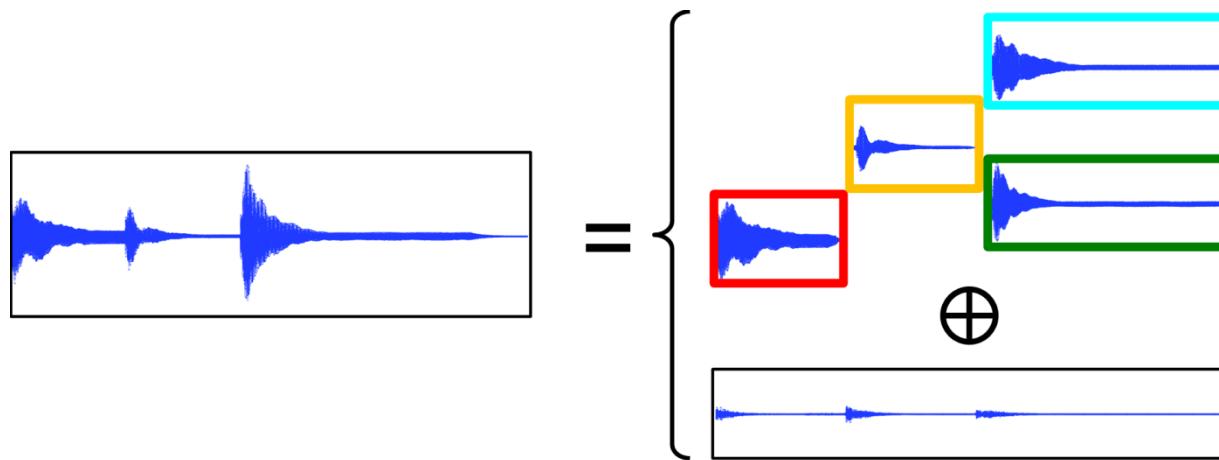
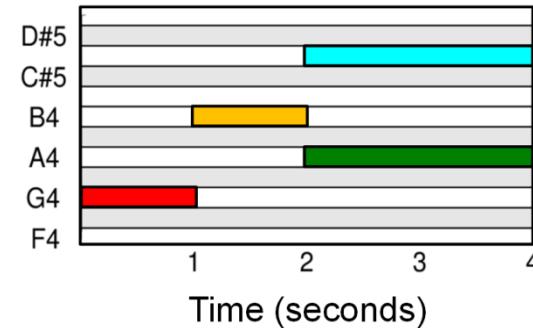
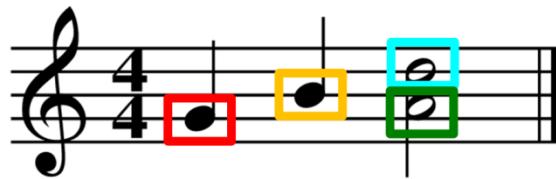
## 8.2 Melody Extraction

Fig. 8.18



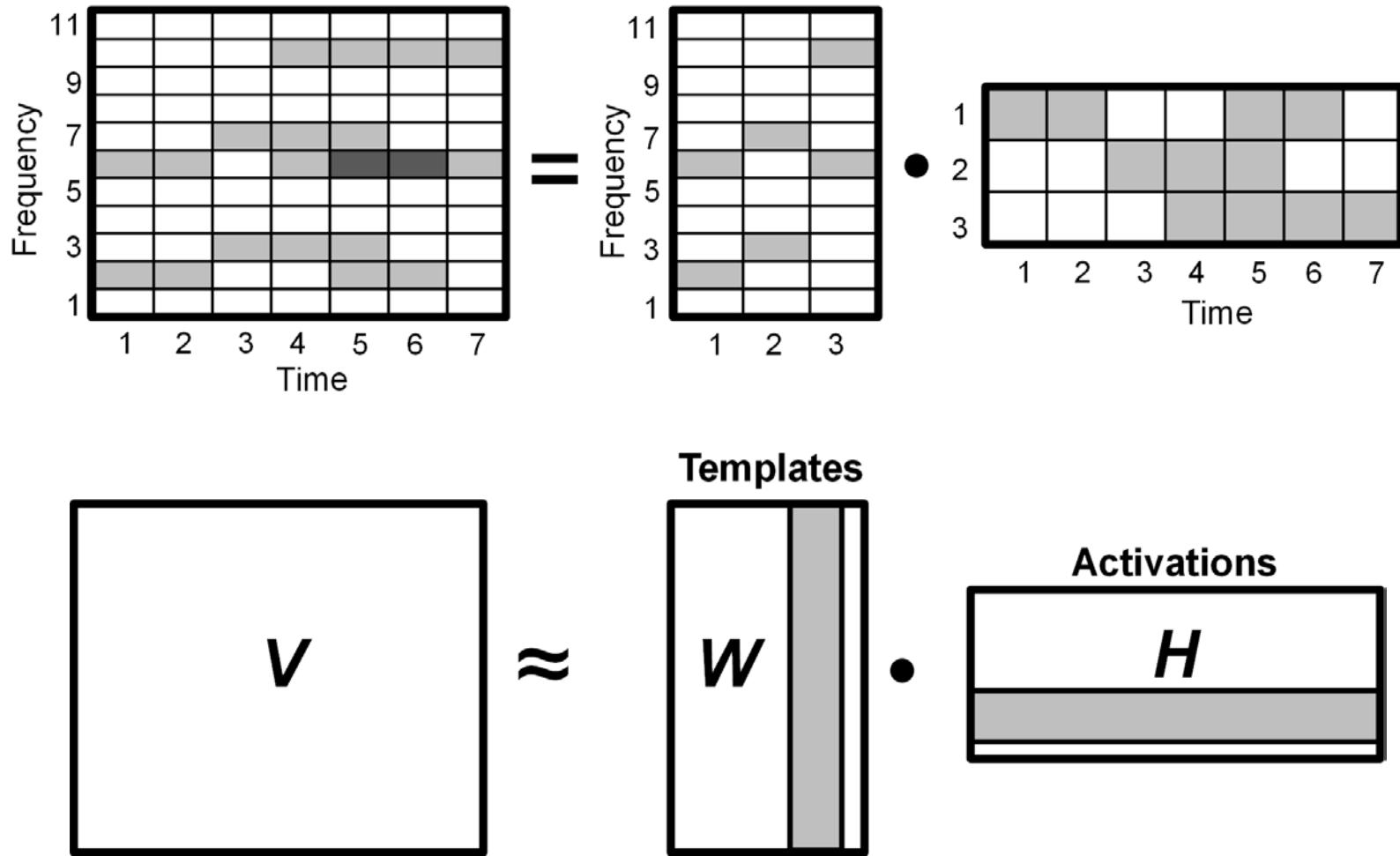
## 8.3 NMF-Based Audio Decomposition

Fig. 8.19



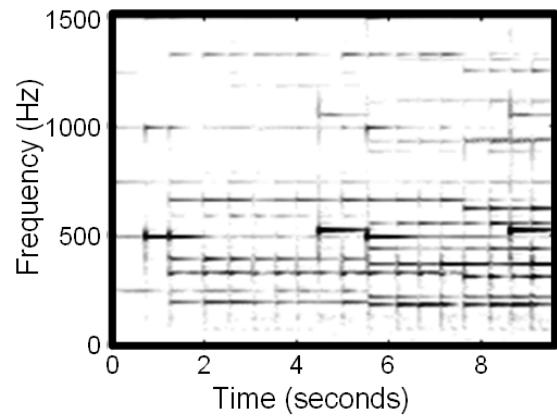
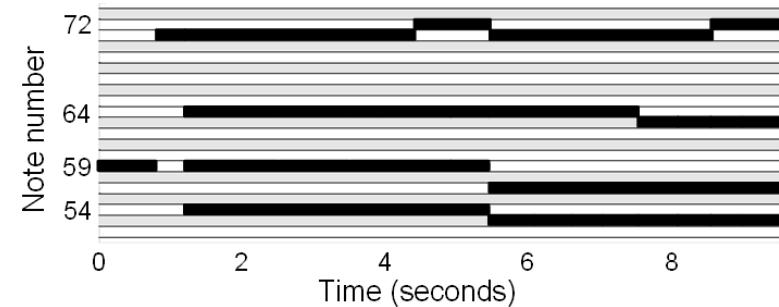
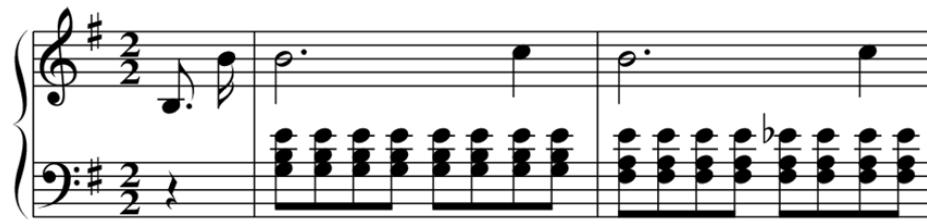
## 8.3 NMF-Based Audio Decomposition

Fig. 8.20

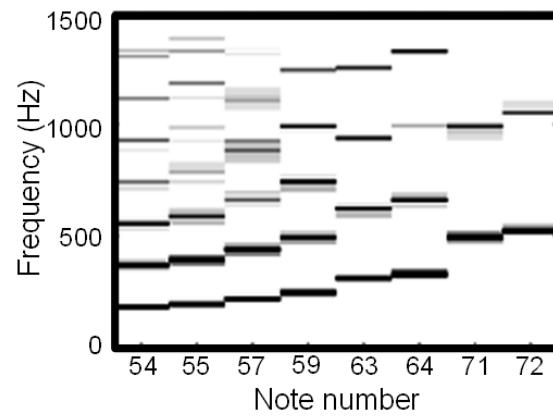


## 8.3 NMF-Based Audio Decomposition

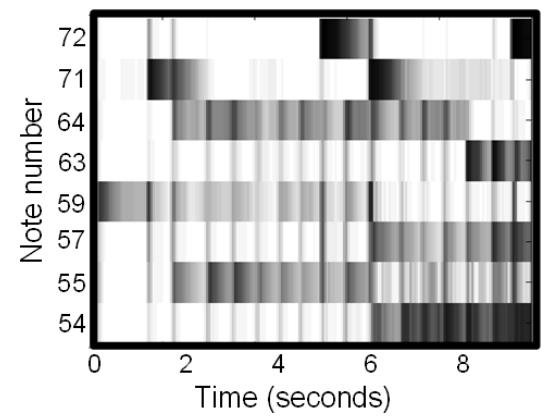
Fig. 8.21



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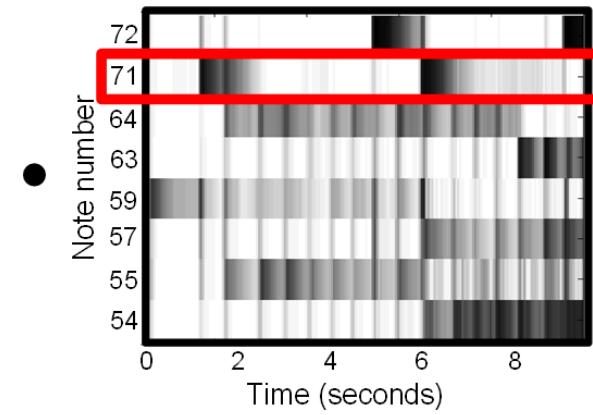
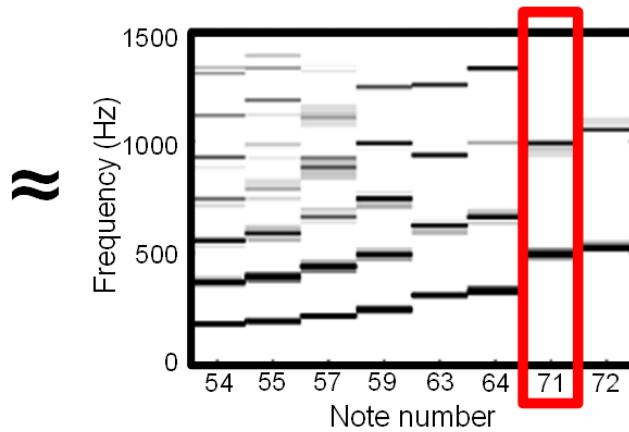
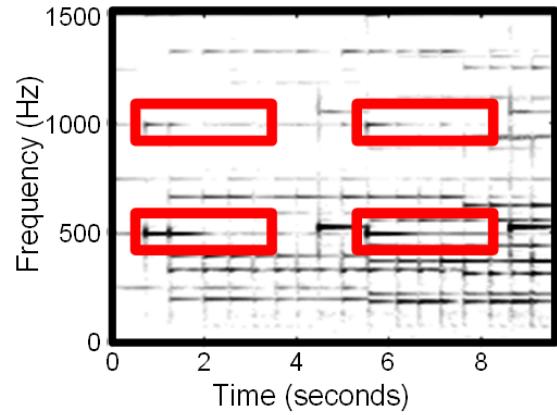
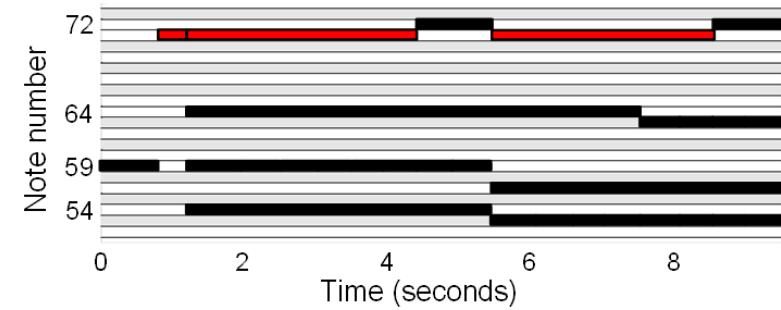
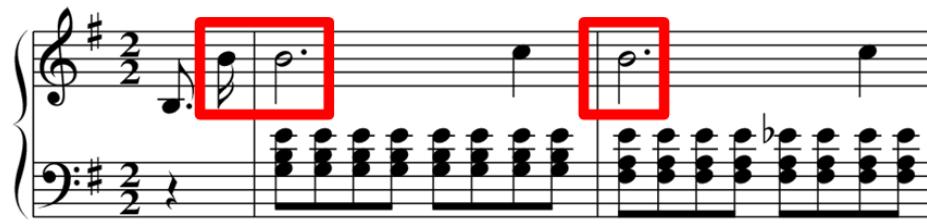


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## 8.3 NMF-Based Audio Decomposition

Fig. 8.21



## 8.3 NMF-Based Audio Decomposition

Table 8.1

**Algorithm:** NMF ( $V \approx WH$ )

**Input:** Nonnegative matrix  $V$  of size  $K \times N$

Rank parameter  $R \in \mathbb{N}$

Threshold  $\varepsilon$  used as stop criterion

**Output:** Nonnegative template matrix  $W$  of size  $K \times R$

Nonnegative activation matrix  $H$  of size  $R \times N$

**Procedure:** Define nonnegative matrices  $W^{(0)}$  and  $H^{(0)}$  by some random or informed initialization. Furthermore set  $\ell = 0$ . Apply the following update rules (written in matrix notation):

$$(1) \quad H^{(\ell+1)} = H^{(\ell)} \odot (((W^{(\ell)})^\top V) \oslash ((W^{(\ell)})^\top W^{(\ell)} H^{(\ell)}))$$

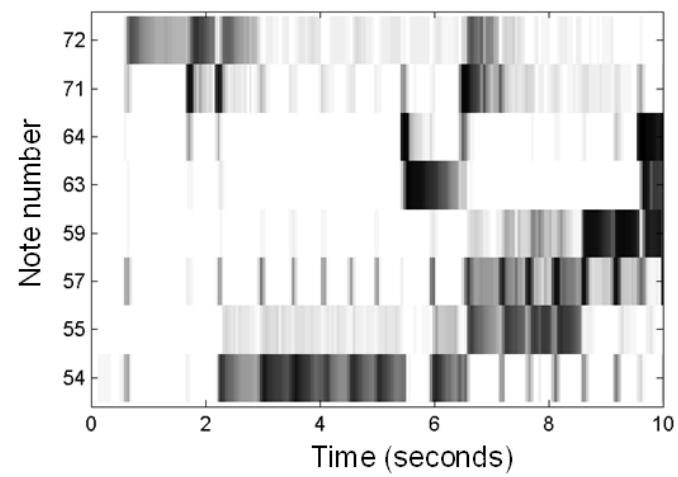
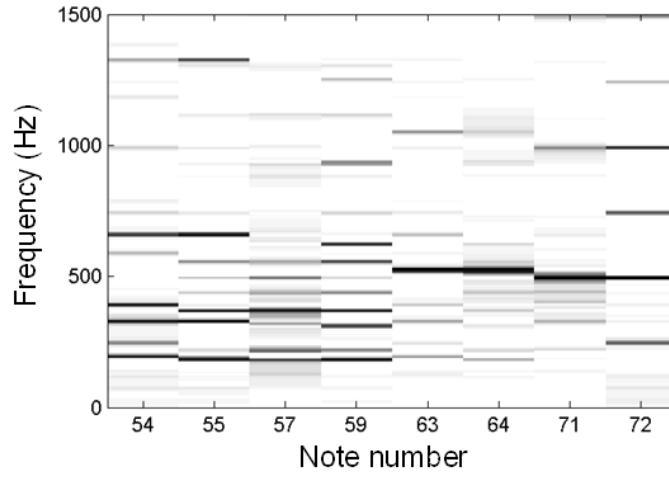
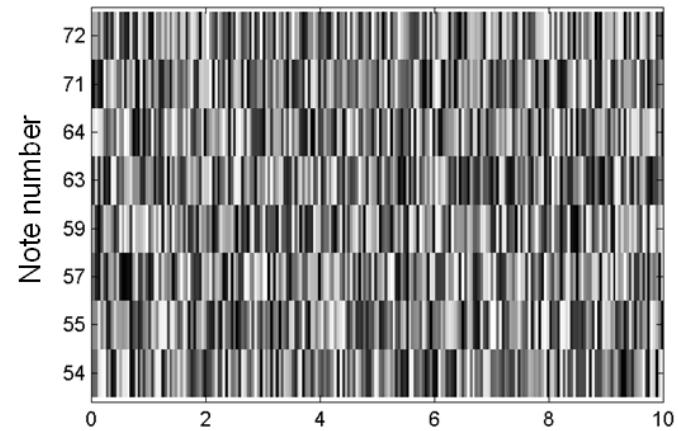
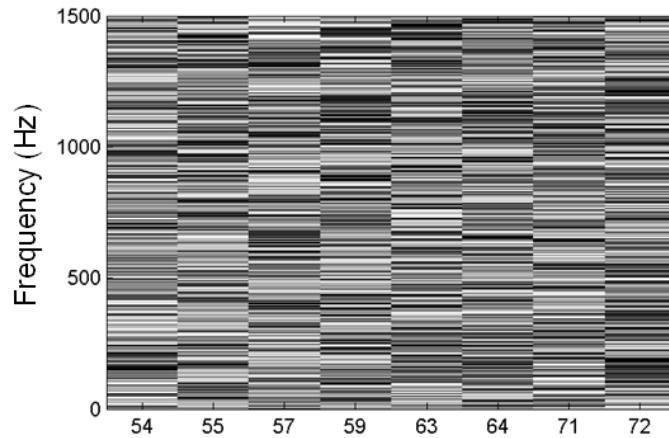
$$(2) \quad W^{(\ell+1)} = W^{(\ell)} \odot ((V(H^{(\ell+1)})^\top) \oslash (W^{(\ell)} H^{(\ell+1)} (H^{(\ell+1)})^\top))$$

(3) Increase  $\ell$  by one.

Repeat the steps (1) to (3) until  $\|H^{(\ell)} - H^{(\ell-1)}\| \leq \varepsilon$  and  $\|W^{(\ell)} - W^{(\ell-1)}\| \leq \varepsilon$  (or until some other stop criterion is fulfilled). Finally, set  $H = H^{(\ell)}$  and  $W = W^{(\ell)}$ .

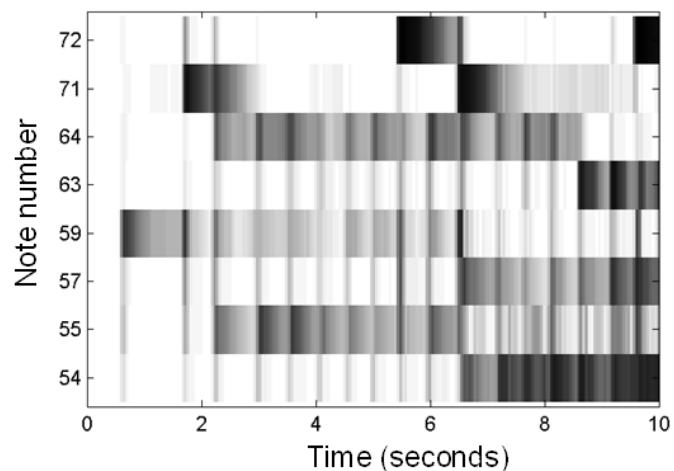
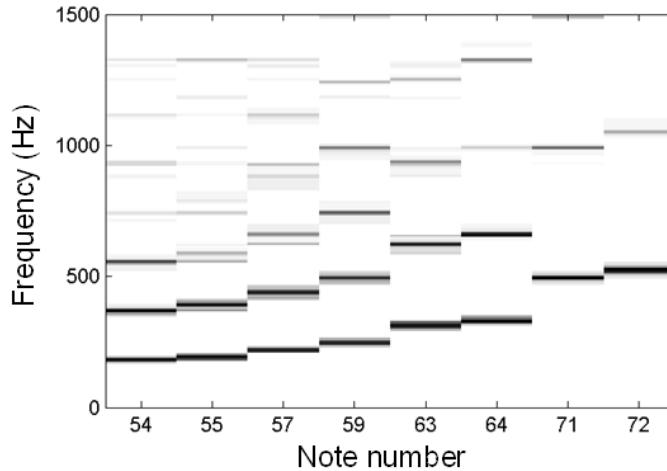
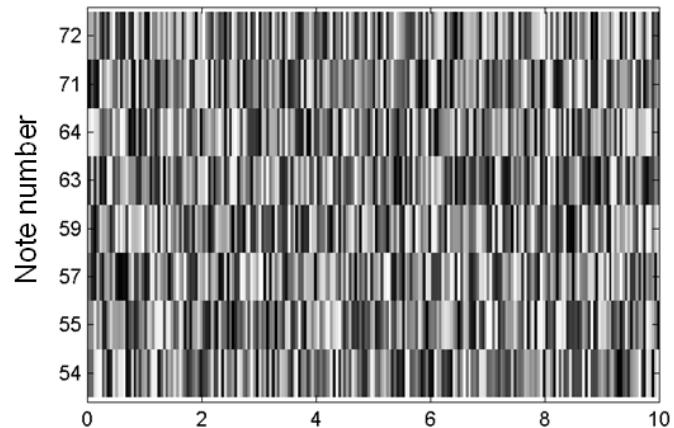
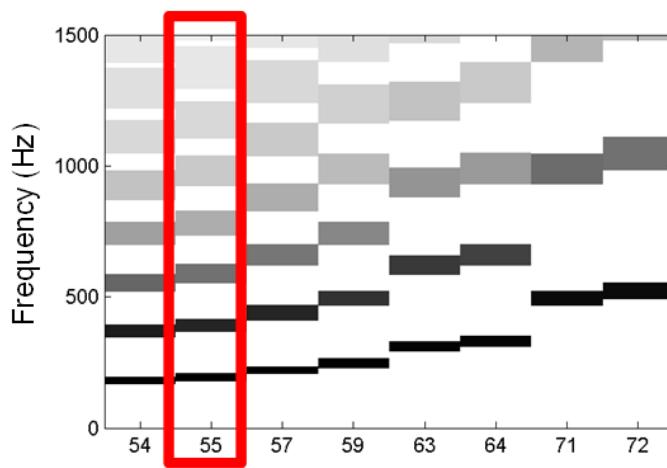
## 8.3 NMF-Based Audio Decomposition

Fig. 8.22



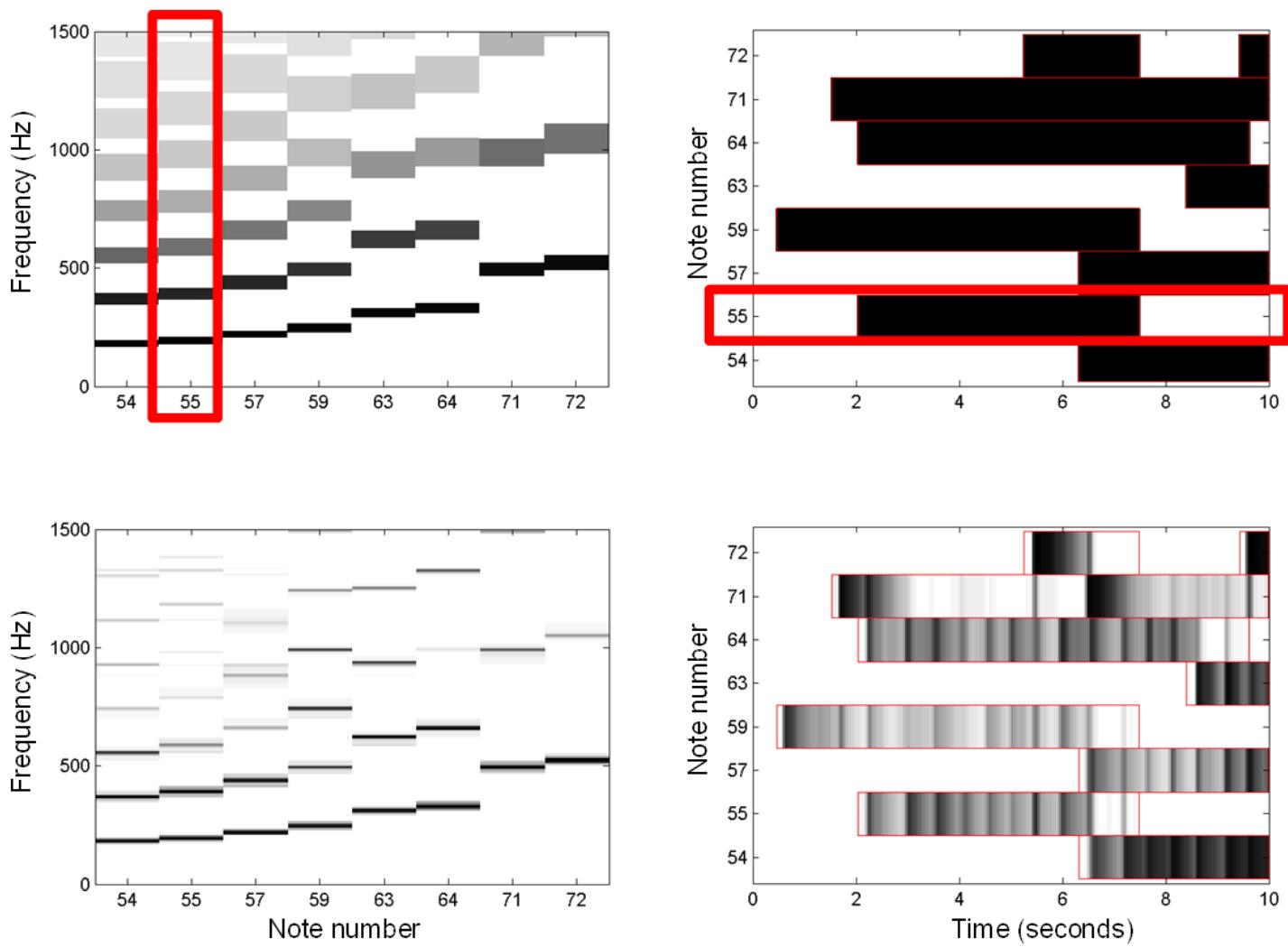
## 8.3 NMF-Based Audio Decomposition

Fig. 8.23



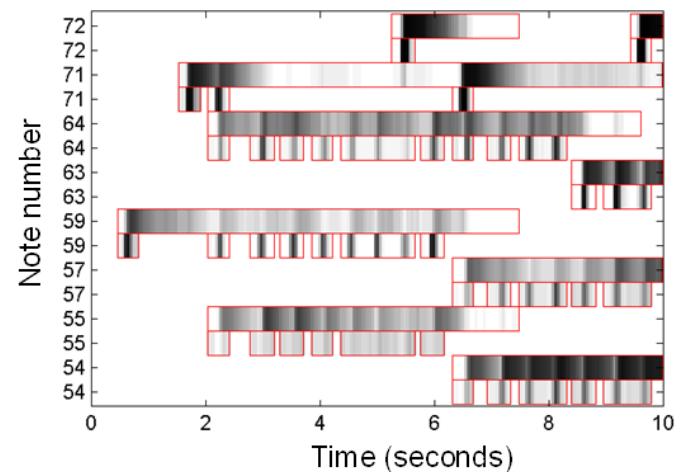
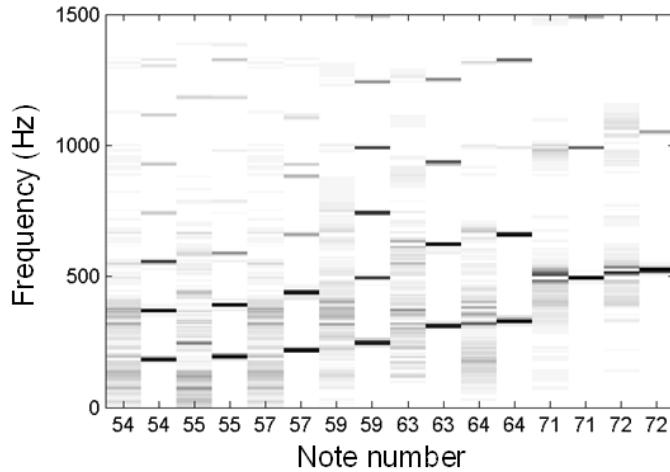
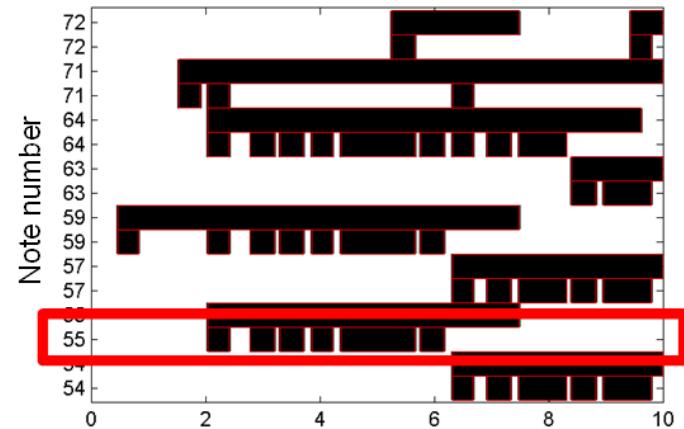
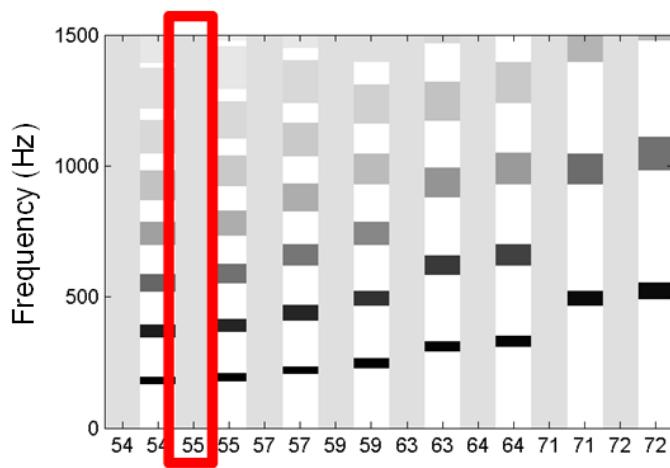
## 8.3 NMF-Based Audio Decomposition

Fig. 8.24



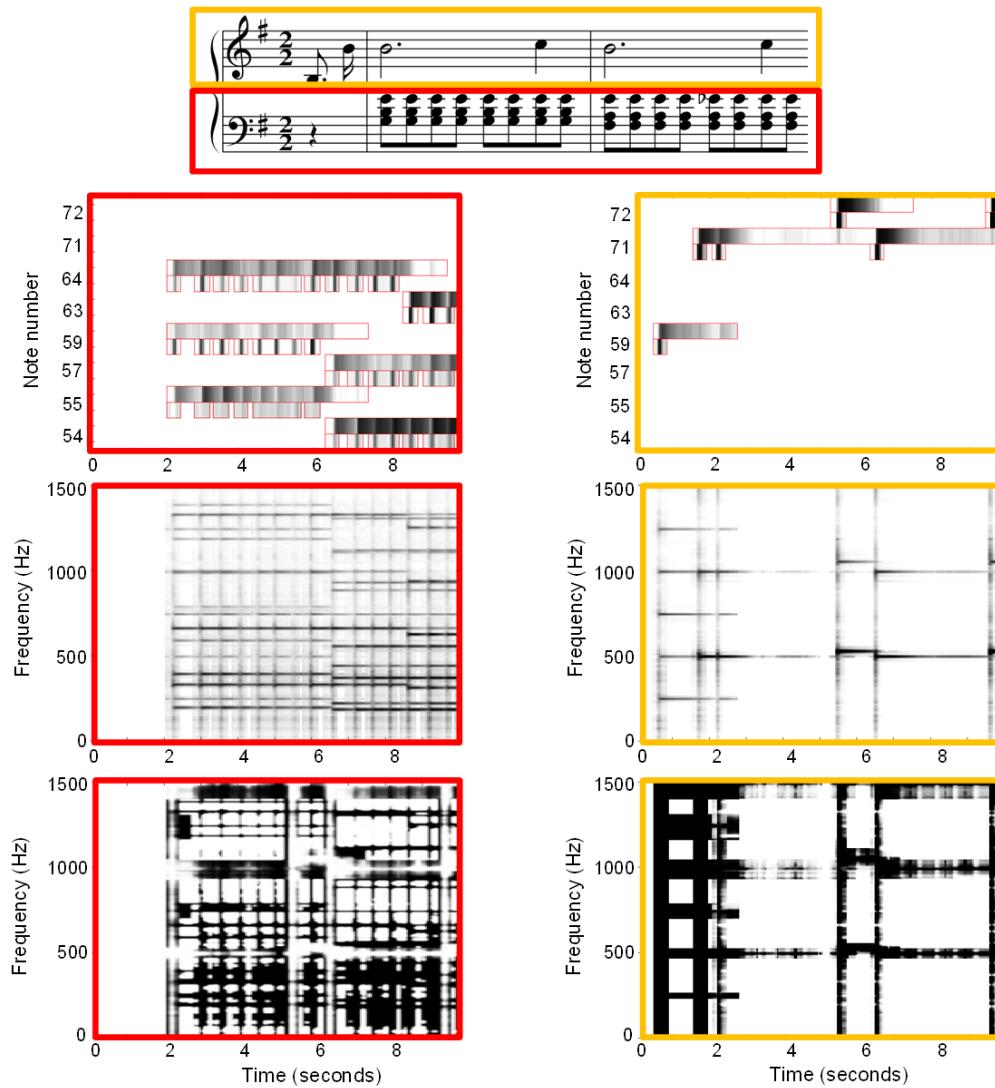
# 8.3 NMF-Based Audio Decomposition

Fig. 8.25



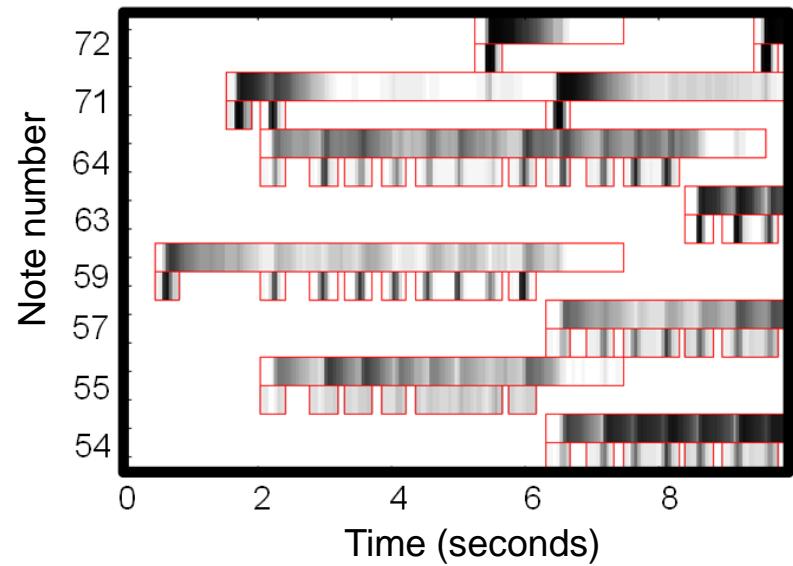
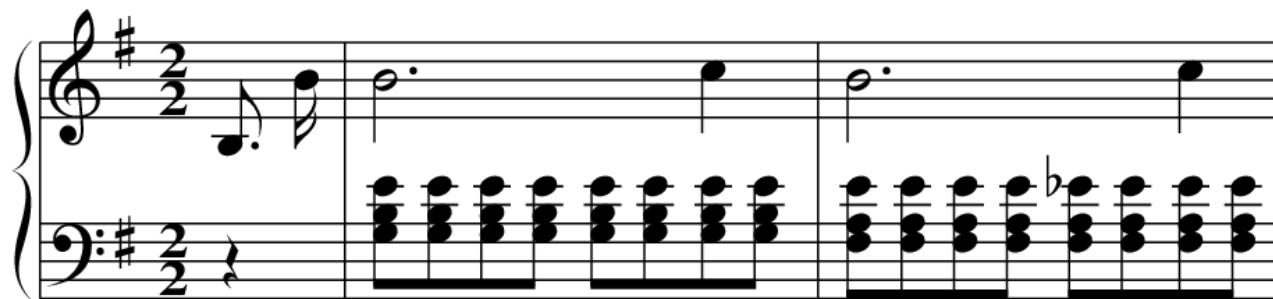
## 8.3 NMF-Based Audio Decomposition

Fig. 8.26



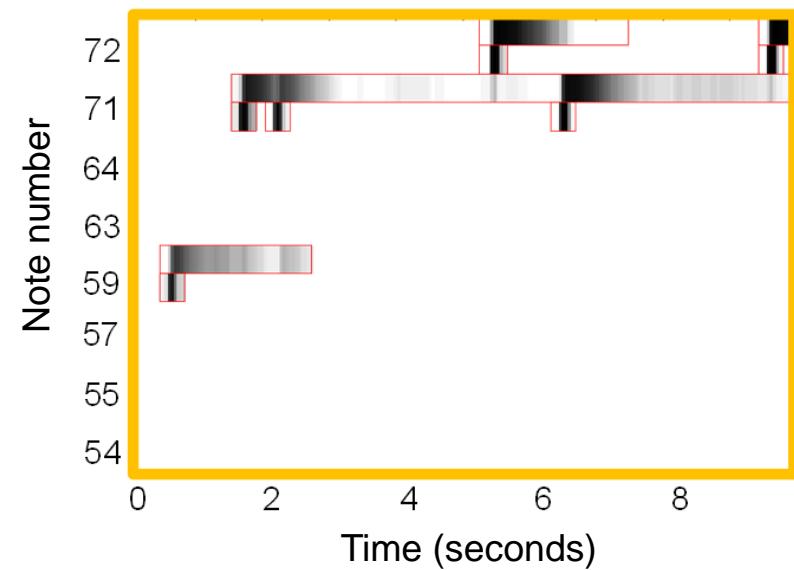
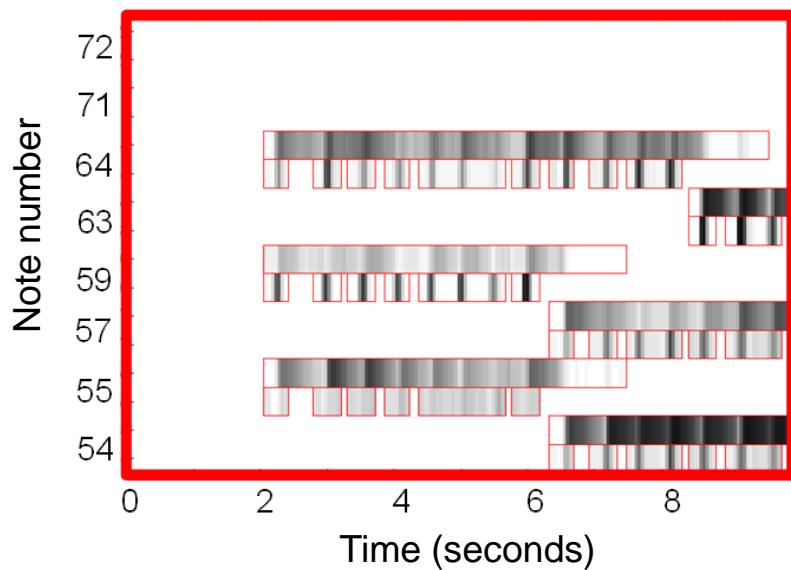
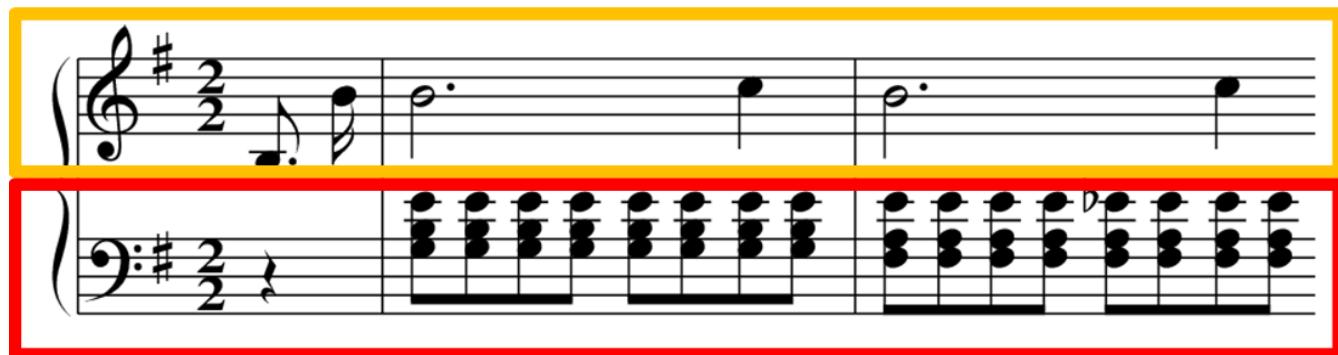
## 8.3 NMF-Based Audio Decomposition

Fig. 8.26



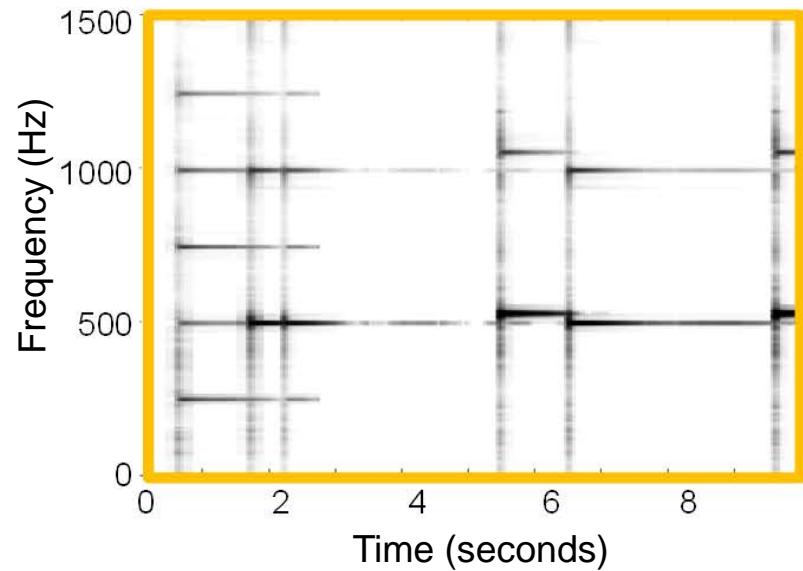
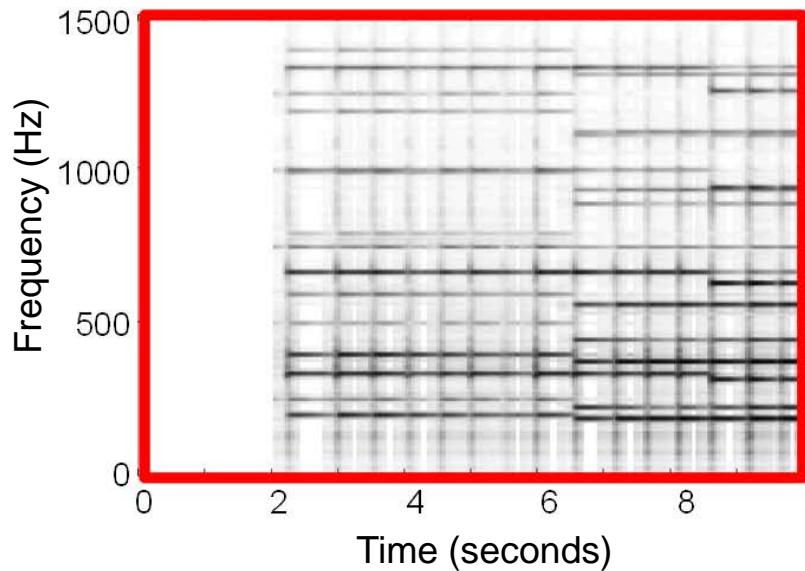
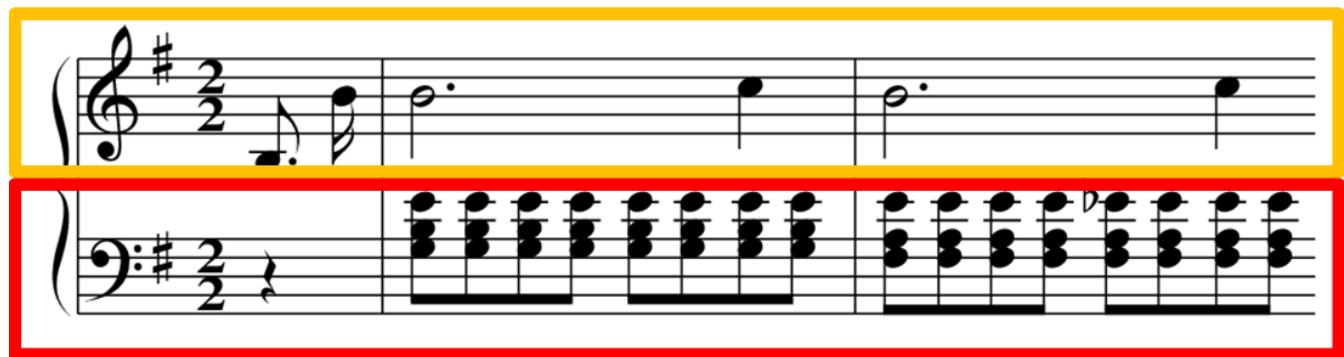
## 8.3 NMF-Based Audio Decomposition

Fig. 8.26



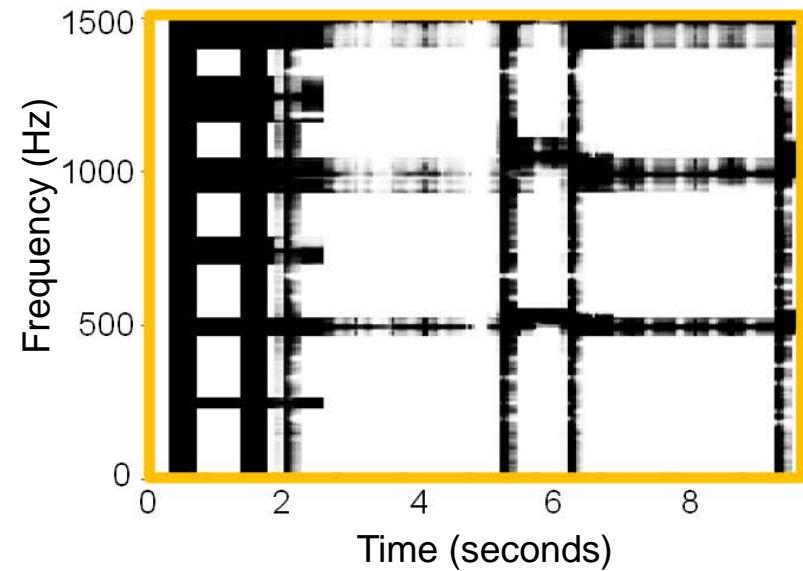
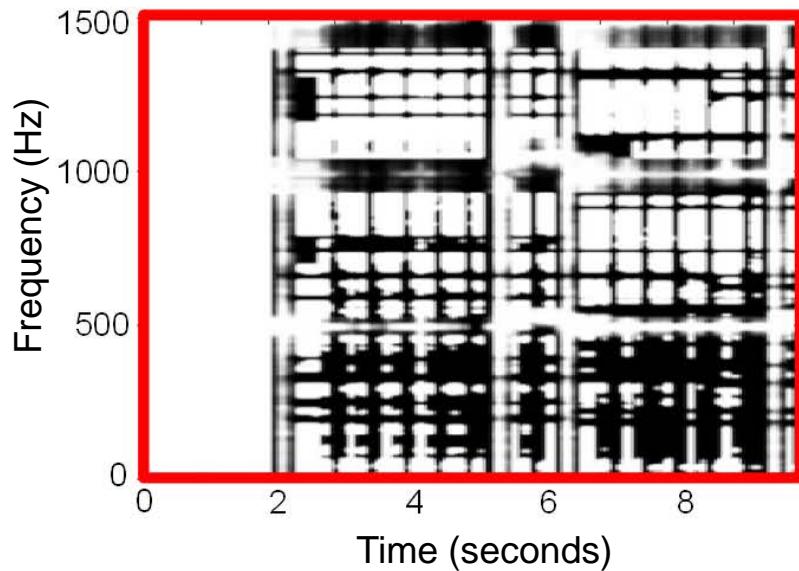
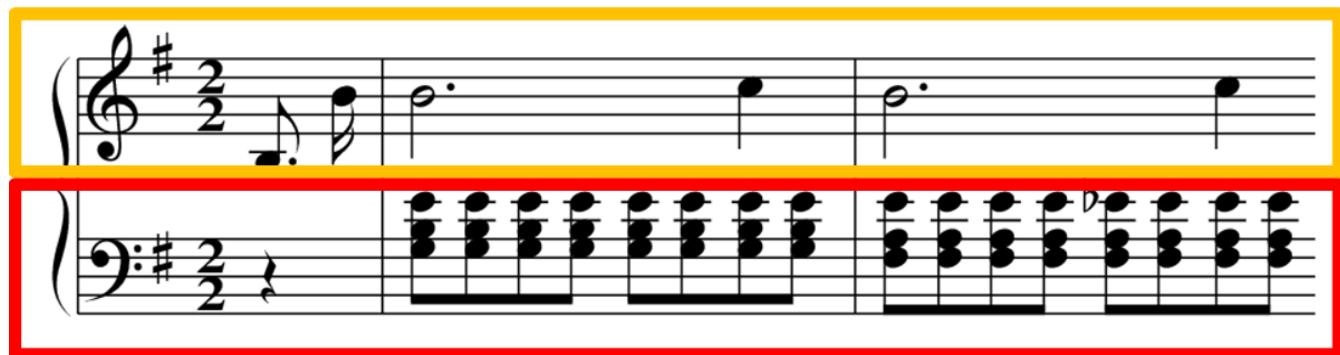
## 8.3 NMF-Based Audio Decomposition

Fig. 8.26



## 8.3 NMF-Based Audio Decomposition

Fig. 8.26



# 8.3 NMF-Based Audio Decomposition

Fig. 8.27

