INTERNATIONAL AUDIO LABORATORIES ERLANGEN

AUDIO LABS

Lecture

Music Processing

Harmony Analysis

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FAU FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

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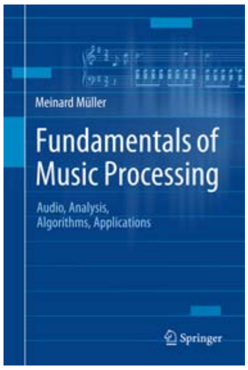
Book: Fundamentals of Music Processing



Meinard Müller
Fundamentals of Music Processing
Audio, Analysis, Algorithms, Applications
483 p., 249 illus., hardcover
ISBN: 978-3-319-21944-8
Springer, 2015

Accompanying website:
www.music-processing.de

Book: Fundamentals of Music Processing

Chapter	Music Processing Scenario
1	Music Represenations
2	Fourier Analysis of Signals
3	Music Synchronization
4	Music Structure Analysis
5	Chord Recognition
6	Tempo and Beat Tracking
7	Content-Based Audio Retrieval
8	Musically Informed Audio Decomposition

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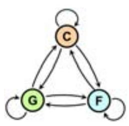
Chapter 5: Chord Recognition

5.1 Basic Theory of Harmony
5.2 Template-Based Chord Recognition
5.3 HMM-Based Chord Recognition
5.4 Further Notes



In Chapter 5, we consider the problem of analyzing harmonic properties of a piece of music by determining a descriptive progression of chords from a given audio recording. We take this opportunity to first discuss some basic theory of harmony including concepts such as intervals, chords, and scales. Then, motivated by the automated chord recognition scenario, we introduce template-based matching procedures and hidden Markov models—a concept of central importance for the analysis of temporal patterns in time-dependent data streams including speech, gestures, and music.

Dissertation: Tonality-Based Style Analysis

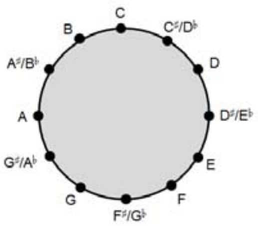
Christof Weiß
Computational Methods for Tonality-Based Style Analysis of Classical Music Audio Recordings
PhD thesis, Ilmenau University of Technology, 2017

Chapter 5: Analysis Methods for Key and Scale Structures
Chapter 6: Design of Tonal Features

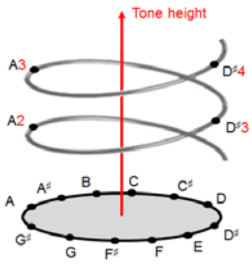
Recall: Chroma Features

- Human perception of pitch is periodic
- Two components: tone height (octave) and chroma (pitch class)

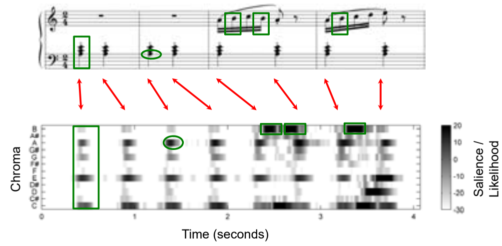
Chromatic circle



Shepard's helix of pitch



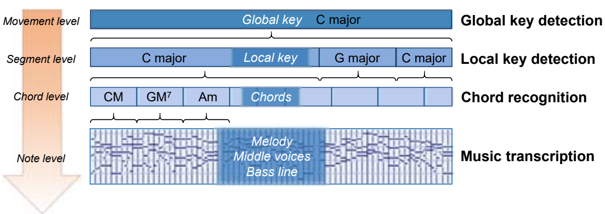
Recall: Chroma Features



→ capture harmonic progression

Harmony Analysis: Overview

- Western music (and most other music): Different aspects of harmony
- Referring to different time scales



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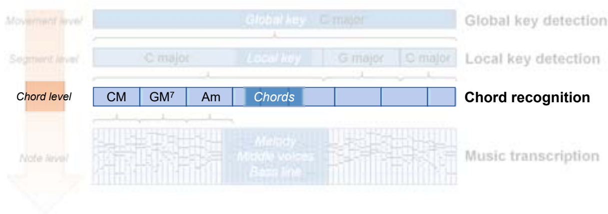
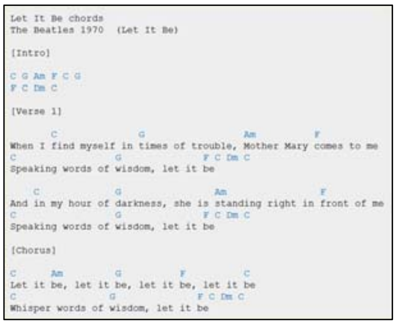


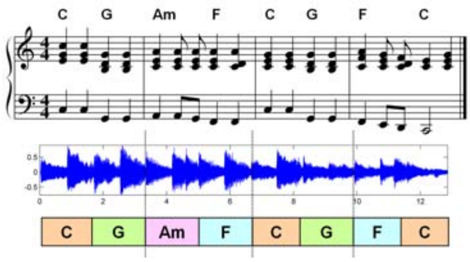
Chord Recognition



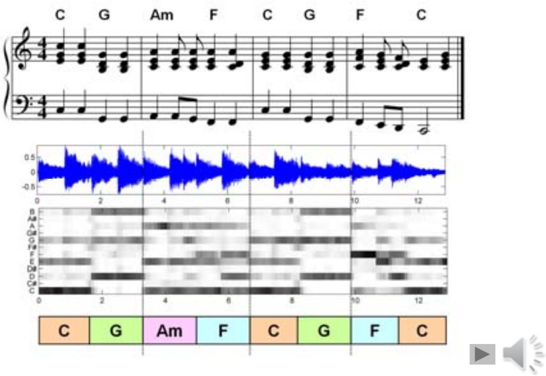
```
Let It Be chords
The Beatles 1970  (Let It Be)

[Intro]

C G Am F C G
F C Dm C

[Verse 1]

      C              G                  Am          F
When I find myself in times of trouble, Mother Mary comes to me
C                G              F C Dm C
Speaking words of wisdom, let it be

   C             G                Am                F
And in my hour of darkness, she is standing right in front of me
C                G              F C Dm C
Speaking words of wisdom, let it be

[Chorus]

C      Am        G         F          C
Let it be, let it be, let it be, let it be
C               G              F C Dm C
Whisper words of wisdom, let it be
```

Source: www.ultimate-guitar.com

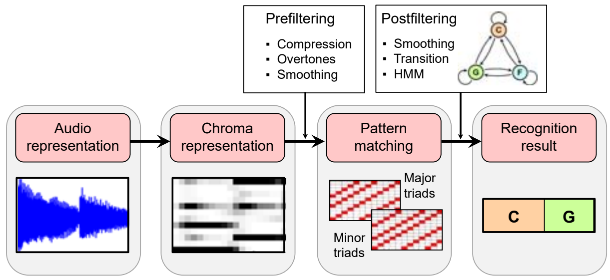
Chord Recognition



Chord Recognition



Chord Recognition



Chord Recognition: Basics

- Musical chord: Group of three or more notes
- Combination of three or more tones which sound simultaneously
- Types: triads (major, minor, diminished, augmented), seventh chords…
- Here: focus on major and minor triads

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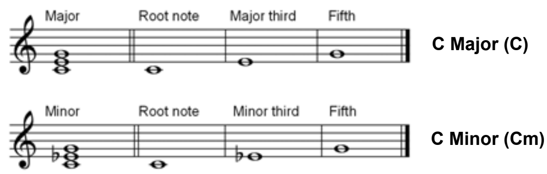


- Enharmonic equivalence: 12 different root notes possible → **24 chords**

Chord Recognition: Basics

Chords appear in different forms:

- Inversions

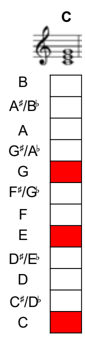
- Different voicings

- Harmonic figuration: Broken chords (arpeggio)

- Melodic figuration: Different melody note (suspension, passing tone, …)
- Further: Additional notes, incomplete chords

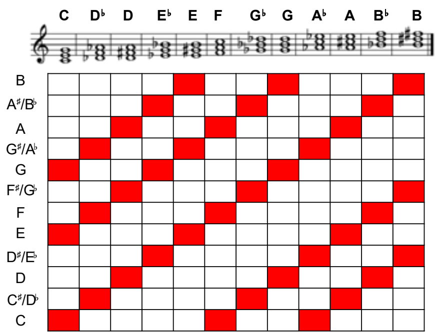
Chord Recognition: Basics

- Templates: **Major Triads**



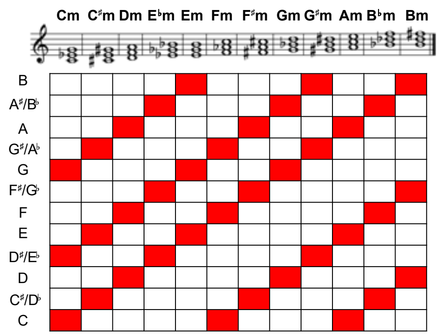
Chord Recognition: Basics

- Templates: **Major Triads**

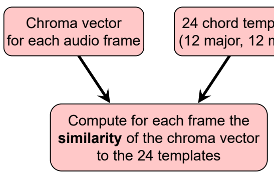


Chord Recognition: Basics

- Templates: **Minor Triads**



Chord Recognition: Template Matching



	C	C♯	D	…	Cm	C♯m	Dm	…
B	0	0	0	…	0	0	0	…
A♯	0	0	0	…	0	0	0	…
A	0	0	1	…	0	0	1	…
G♯	0	1	0	…	0	1	0	…
G	1	0	0	…	1	0	0	…
F♯	0	0	1	…	0	0	0	…
F	0	1	0	…	0	0	1	…
E	1	0	0	…	0	1	0	…
D♯	0	0	0	…	1	0	0	…
D	0	0	1	…	0	0	1	…
C♯	0	1	0	…	0	1	0	…
C	1	0	0	…	1	0	0	…

Chord Recognition: Template Matching

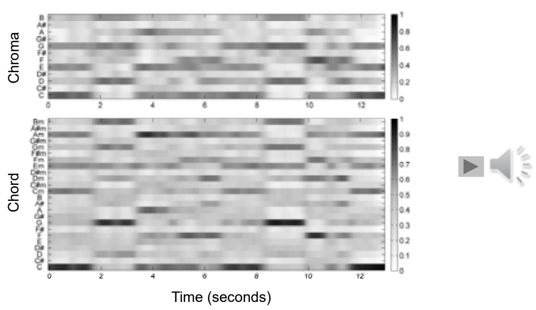
- Similarity measure: Cosine similarity (inner product of normalized vectors)

Chord template: $\boldsymbol{t} \in \mathbb{R}^{12}$

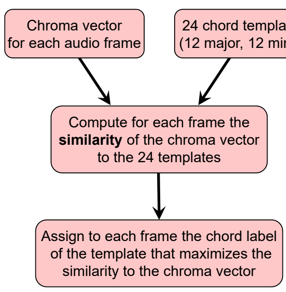
Chroma vector: $\boldsymbol{c} \in \mathbb{R}^{12}$

Similarity measure: $s(\boldsymbol{t},\boldsymbol{c}) = \dfrac{\langle \boldsymbol{t}|\boldsymbol{c}\rangle}{||\boldsymbol{t}|| \cdot ||\boldsymbol{c}||}$

Chord Recognition: Template Matching

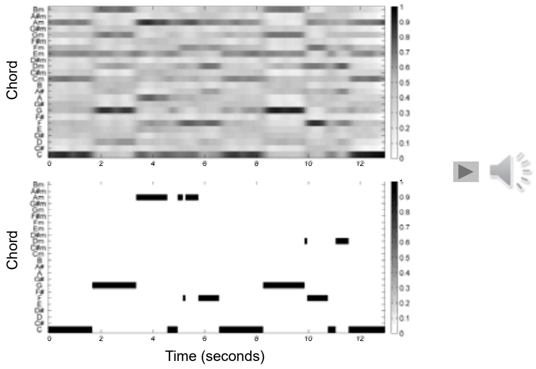


Chord Recognition: Label Assignment

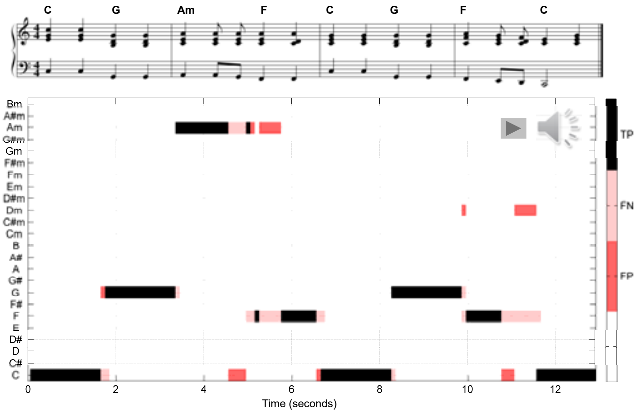


	C	C♯	D	…	Cm	C♯m	Dm	…
B	0	0	0	…	0	0	0	…
A♯	0	0	0	…	0	0	0	…
A	0	0	1	…	0	0	1	…
G♯	0	1	0	…	0	1	0	…
G	1	0	0	…	1	0	0	…
F♯	0	0	1	…	0	0	0	…
F	0	1	0	…	0	0	1	…
E	1	0	0	…	0	1	0	…
D♯	0	0	0	…	1	0	0	…
D	0	0	1	…	0	0	1	…
C♯	0	1	0	…	0	1	0	…
C	1	0	0	…	1	0	0	…

Chord Recognition: Label Assignment



Chord Recognition: Evaluation

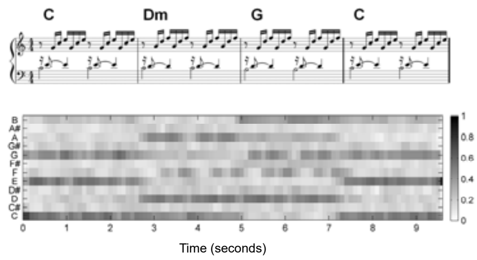


Chord Recognition: Evaluation

- “No-Chord” annotations: not every frame labeled
- Different evaluation measures:
  - Precision: $$P = \frac{\#\mathrm{TP}}{\#\mathrm{TP} + \#\mathrm{FP}}$$
  - Recall: $$R = \frac{\#\mathrm{TP}}{\#\mathrm{TP} + \#\mathrm{FN}}$$
  - F-Measure (balances precision and recall): $$F = \frac{2 \cdot P \cdot R}{P + R}$$
- Without “No-Chord” label: $P = R = F$

Chord Recognition: Smoothing

- Apply average filter of length $L \in \mathbb{N}$:



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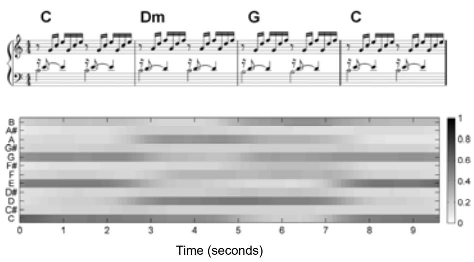
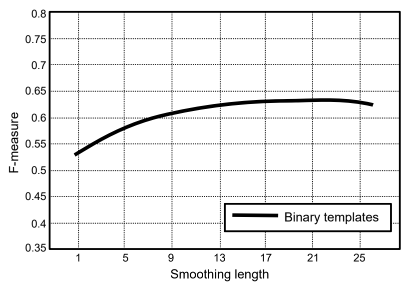


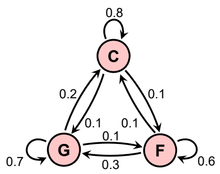
Chord Recognition: Smoothing

- Evaluation on all Beatles songs



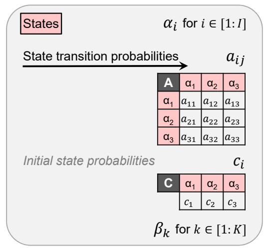
Markov Chains

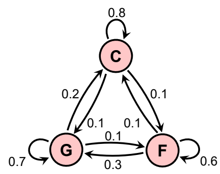
- Probabilistic model for sequential data
- Markov property: Next state only depends on current state (no "memory")
- Consist of:
  - Set of states (hidden)
  - State transition probabilities
  - *Initial state probabilities*



Markov Chains

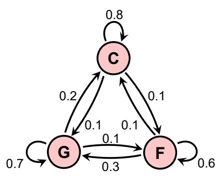
Notation:





Markov Chains

- Application examples:
  - Compute probability of a sequence using given a model (evaluation)
  - Compare two sequences using a given model
  - Evaluate a sequence with two different models (classification)



Hidden Markov Models

- States as **hidden** variables
- Consist of:
  - Set of states (hidden)
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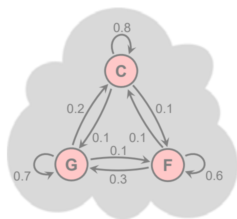

Hidden Markov Models

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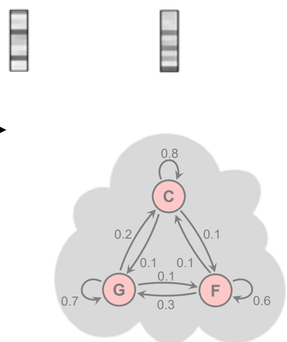
  - Observations (visible)



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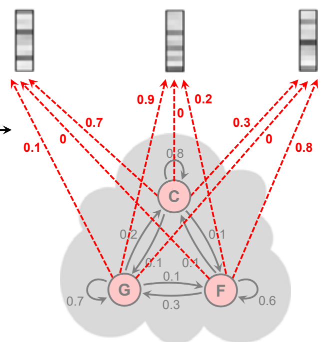
  - Observations (visible)
  - Emission probabilities



Hidden Markov Models

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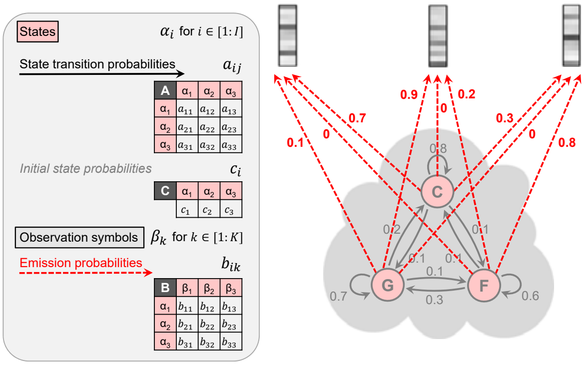

Markov Chains

- Analogon: the student's life
- Consists of:
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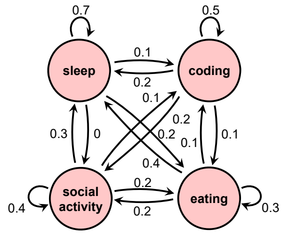

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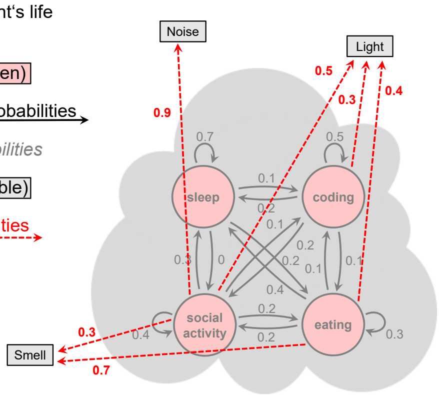


Hidden Markov Models

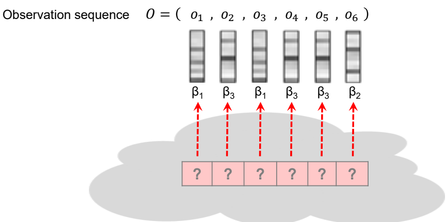
- Only observation sequence is visible!

Different algorithmic problems:

- **Evaluation problem**
  - Given: observation sequence and model
  - Calculate how well the model matches the sequence
- **Uncovering problem:**
  - Given: observation sequence and model
  - Find: optimal hidden state sequence
- **Estimation problem** („training" the HMM):
  - Given: observation sequence
  - Find: model parameters
  - Baum-Welch algorithm (Expectation-Maximization)

Uncovering problem

- Given: observation sequence $O = (o_1, \dots, o_N)$ of length $N \in \mathbb{N}$ and HMM θ (model parameters)
- Find: optimal hidden state sequence $S^* = (s_1^*, \dots, s_N^*)$
- Corresponds to chord estimation task!



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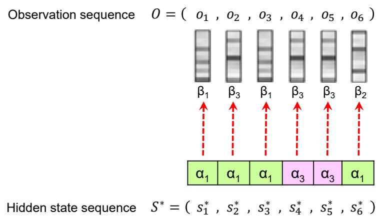

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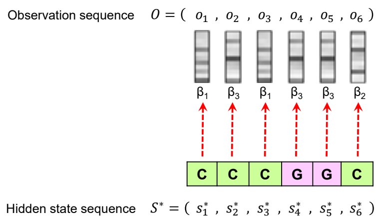

Uncovering problem

- **Optimal** hidden state sequence?
  - "Best explains" given observation sequence O
  - Maximizes probability $P[O, S \mid \Theta]$

$$\mathrm{Prob}^* = \max_S P[O, S \mid \Theta]$$

$$S^* = \operatorname{argmax}_S P[O, S \mid \Theta]$$

- Straight-forward computation (naive approach):
  - Compute probability for each possible sequence S
  - Number of possible sequences of length N (I = number of states):

$$\underbrace{I \cdot I \cdot \ldots \cdot I}_{N \text{ factors}} = I^N$$

computationally infeasible!

Viterbi Algorithm

- Based on dynamic programming (similar to DTW)
- Idea: Recursive computation from subproblems
- Use truncated versions of observation sequence

$$O(1{:}n) := (o_1, \ldots, o_n), \text{ length } n \in [1{:}N]$$

- Define $\mathbf{D}(i, n)$ as the highest probability along a single state sequence $(s_1, \ldots, s_n)$ that ends in state $s_n = \alpha_i$

$$\mathbf{D}(i, n) = \max_{(s_1, \ldots, s_n)} P[O(1{:}n), (s_1, \ldots, s_{n-1}, s_n = \alpha_i) \mid \Theta]$$

- Then, our solution is the state sequence yielding

$$\mathrm{Prob}^* = \max_{i \in [1:I]} \mathbf{D}(i, N)$$

Viterbi Algorithm

- $\mathbf{D}$: matrix of size $I \times N$
- Recursive computation of $\mathbf{D}(i, n)$ along the column index n
- Initialization:
  - $n = 1$
  - Truncated observation sequence: $O(1) = (o_1)$
  - Current observation: $o_1 = \beta_{k_1}$

$$\mathbf{D}(i, 1) = c_i \cdot b_{ik_1} \quad \text{for some } i \in [1{:}I]$$

Viterbi Algorithm

- $\mathbf{D}$: matrix of size $I \times N$
- Recursive computation of $\mathbf{D}(i, n)$ along the column index n
- Recursion:
  - $n \in [2{:}N]$
  - Truncated observation sequence: $O(1{:}n) = (o_1, \ldots, o_n)$
  - Last observation: $o_n = \beta_{k_n}$

$$\mathbf{D}(i, n) = b_{ik_n} \cdot a_{j^*i} \cdot \underbrace{P\big[O(1{:}n-1), (s_1, \ldots, s_{n-1} = \alpha_{j^*}) \,\big|\, \Theta\big]}_{\text{must be maximal!}} \quad \text{for } i \in [1{:}I]$$

$$\mathbf{D}(i, n) = b_{ik_n} \cdot a_{j^*i} \cdot \mathbf{D}(j^*, n-1)$$

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$$\mathbf{D}(i, n) = b_{ik_n} \cdot \underbrace{a_{j^*i} \cdot \mathbf{D}(j^*, n-1)}_{\text{must be maximal (best index } j^*)}$$

$$\mathbf{D}(i, n) = b_{ik_n} \cdot \max_{j \in [1:I]} \big(a_{ji} \cdot \mathbf{D}(j, n-1)\big)$$

Viterbi Algorithm

- $\mathbf{D}$ given – find optimal state sequence $S^* = (s_1^*, \dots, s_N^*) := (\alpha_{i_1}, \dots, \alpha_{i_N})$
- Backtracking procedure (reverse order)
- Last element:
  - $n = N$
  - Optimal state: α_{i_N}

$$i_N = \underset{j \in [1:I]}{\operatorname{argmax}} \, \mathbf{D}(j, N)$$

Viterbi Algorithm

- $\mathbf{D}$ given – find optimal state sequence $S^* = (s_1^*, \dots, s_N^*) := (\alpha_{i_1}, \dots, \alpha_{i_N})$
- Backtracking procedure (reverse order)
- Further elements:
  - $n = N-1, N-2, \dots, 1$
  - Optimal state: α_{i_n}

$$i_n = \underset{j \in [1:I]}{\operatorname{argmax}} \left(a_{j i_{n+1}} \cdot \mathbf{D}(j, n) \right)$$

Viterbi Algorithm

- $\mathbf{D}$ given – find optimal state sequence $S^* = (s_1^*, \dots, s_N^*) := (\alpha_{i_1}, \dots, \alpha_{i_N})$
- Backtracking procedure (reverse order)
- Further elements:
  - $n = N-1, N-2, \dots, 1$
  - Optimal state: α_{i_n}

$$i_n = \underset{j \in [1:I]}{\operatorname{argmax}} \left(a_{j i_{n+1}} \cdot \mathbf{D}(j, n) \right)$$

- Simplification of backtracking: Keep track of maximizing index j in

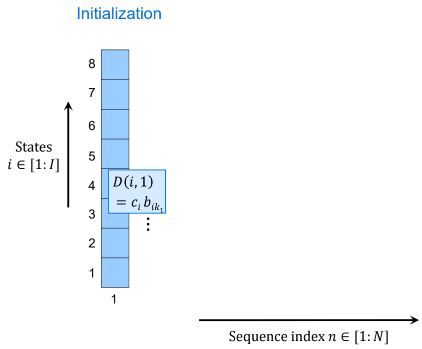
$$\mathbf{D}(i, n) = b_{i k_n} \cdot \max_{j \in [1:I]} \left(a_{ji} \cdot \mathbf{D}(j, n-1) \right)$$

- Define $(I \times (N-1))$ matrix $\mathbf{E}$:

$$\mathbf{E}(i, n-1) = \underset{j \in [1:I]}{\operatorname{argmax}} \left(a_{ji} \cdot \mathbf{D}(j, n-1) \right)$$

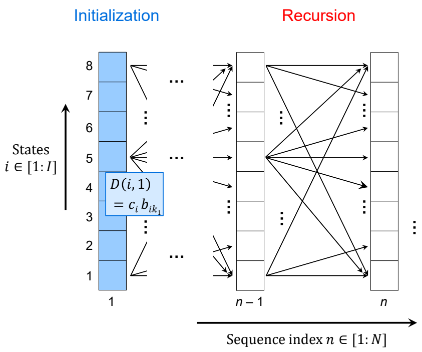
Viterbi Algorithm

Summary



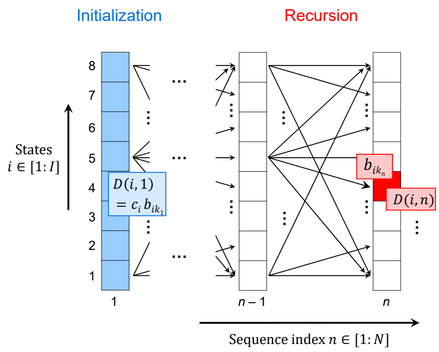
Viterbi Algorithm

Summary



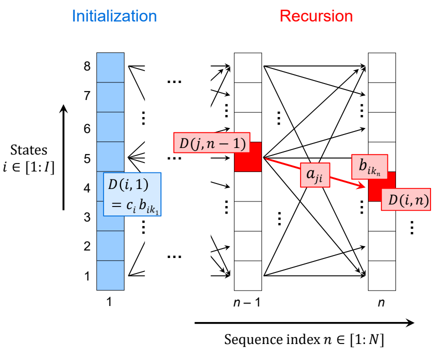
Viterbi Algorithm

Summary



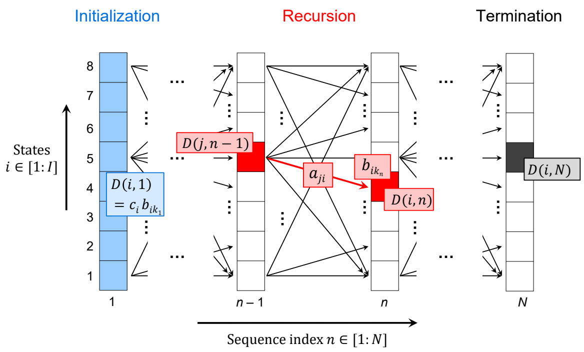
Viterbi Algorithm

Summary



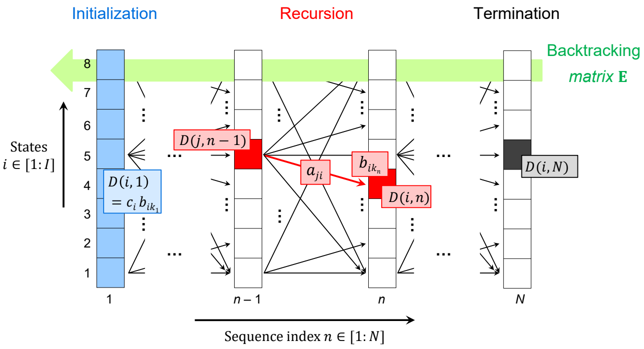
Viterbi Algorithm

Summary



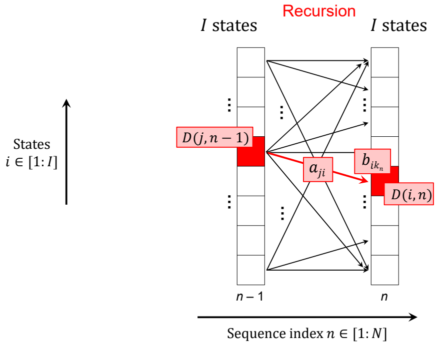
Viterbi Algorithm

Summary



Viterbi Algorithm

Computational Complexity



Per recursion step:
$I \cdot I$

Total recursion:
$I^2 \cdot N$

Viterbi Algorithm

Summary

Algorithm: VITERBI

Input: HMM specified by $\Theta = (\mathcal{A}, A, C, \mathcal{B}, B)$
Observation sequence $O = (o_1 = \beta_{k_1}, o_2 = \beta_{k_2}, \ldots, o_N = \beta_{k_N})$

Output: Optimal state sequence $S^* = (s_1^*, s_2^*, \ldots, s_N^*)$

Procedure: Initialize the $(I \times N)$ matrix $\mathbf{D}$ by $\mathbf{D}(i,1) = c_i b_{ik_1}$ for $i \in [1:I]$. Then compute in a nested loop for $n = 2, \ldots, N$ and $i = 1, \ldots, I$:

$$\mathbf{D}(i,n) = \max_{j \in [1:I]} \left(a_{ji} \cdot \mathbf{D}(j, n-1)\right) \cdot b_{ik_n}$$
$$\mathbf{E}(i,n-1) = \operatorname{argmax}_{j \in [1:I]} \left(a_{ji} \cdot \mathbf{D}(j, n-1)\right)$$

Set $i_N = \operatorname{argmax}_{j \in [1:I]} \mathbf{D}(j,N)$ and compute for decreasing $n = N-1, \ldots, 1$ the maximizing indices

$$i_n = \operatorname{argmax}_{j \in [1:I]} \left(a_{ji_{n+1}} \cdot \mathbf{D}(j,n)\right) = \mathbf{E}(i_{n+1}, n).$$

The optimal state sequence $S^* = (s_1^*, \ldots, s_N^*)$ is defined by $s_n^* = \alpha_{i_n}$ for $n \in [1:N]$.

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	a_{11}	a_{12}	a_{13}
α_2	a_{21}	a_{22}	a_{23}
α_3	a_{31}	a_{32}	a_{33}

Emission probabilities
b_{ik}

B	β_1	β_2	β_3
α_1	b_{11}	b_{12}	b_{13}
α_2	b_{21}	b_{22}	b_{23}
α_3	b_{31}	b_{32}	b_{33}

Initial state probabilities
c_i

C	α_1	α_2	α_3
	c_1	c_2	c_3

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities
b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities
b_{ik}

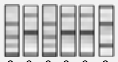
B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

Observation sequence
O = (o_1,o_2,o_3,o_4,o_5,o_6)



β_1 β_3 β_1 β_3 β_3 β_2

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities
b_{ik}

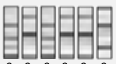
B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

Observation sequence
O = (o_1,o_2,o_3,o_4,o_5,o_6)



β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1						
α_2						
α_3						

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1					
α_2					
α_3					

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
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α_3	0.1	0.3	0.6

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b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6
β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1						
α_2						
α_3						

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1					
α_2					
α_3					

Initialization

$$\mathbf{D}(i,1) = c_i \cdot b_{ik_1}$$

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities
b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6
β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1	0.4200					
α_2	0.0200					
α_3	0					

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1					
α_2					
α_3					

Initialization

$$\mathbf{D}(i,1) = c_i \cdot b_{ik_1}$$

Recursion

$$\mathbf{D}(i,n) = b_{ik_n} \cdot \max_{j\in[1:I]} \left(a_{ji} \cdot \mathbf{D}(j,n-1)\right)$$

$$\mathbf{E}(i,n-1) = \operatorname*{argmax}_{j\in[1:I]} \left(a_{ji} \cdot \mathbf{D}(j,n-1)\right)$$

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities
b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

o_1 o_2 o_3 o_4 o_5 o_6
β_1 β_3 β_1 β_3 β_3 β_2

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1	0.4200	0.1008				
α_2	0.0200	0				
α_3	0	0.0336				

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1	1				
α_2	1				
α_3	1				

Initialization

$$\mathbf{D}(i,1) = c_i \cdot b_{ik_1}$$

Recursion

$$\mathbf{D}(i,n) = b_{ik_n} \cdot \max_{j\in[1:I]} \left(a_{ji} \cdot \mathbf{D}(j,n-1)\right)$$

$$\mathbf{E}(i,n-1) = \operatorname*{argmax}_{j\in[1:I]} \left(a_{ji} \cdot \mathbf{D}(j,n-1)\right)$$

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities
b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

$o_1\ o_2\ o_3\ o_4\ o_5\ o_6$
$\beta_1\ \beta_3\ \beta_1\ \beta_3\ \beta_3\ \beta_2$

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1	0.4200	0.1008	0.0564	0.0135	0.0033	0
α_2	0.0200	0	0.0010	0	0	0.0006
α_3	0	0.0336	0	0.0045	0.0022	0.0003

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1	1	1	1	1	1
α_2	1	1	1	1	3
α_3	1	3	1	3	3

Backtracking

$$i_N = \operatorname*{argmax}_{j\in[1:I]} \mathbf{D}(j,n)$$

$$i_n = \mathbf{E}(i_{n+1},n)$$

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
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α_3	0.1	0.3	0.6

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b_{ik}

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

$o_1\ o_2\ o_3\ o_4\ o_5\ o_6$
$\beta_1\ \beta_3\ \beta_1\ \beta_3\ \beta_3\ \beta_2$

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1	0.4200	0.1008	0.0564	0.0135	0.0033	0
α_2	0.0200	0	0.0010	0	0	0.0006
α_3	0	0.0336	0	0.0045	0.0022	0.0003

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1	1	1	1	1	1
α_2	1	1	1	1	3
α_3	1	3	1	3	3

Backtracking

$$i_N = \operatorname*{argmax}_{j\in[1:I]} \mathbf{D}(j,n)$$

$$i_n = \mathbf{E}(i_{n+1},n)$$

Viterbi Algorithm: Example

HMM:

States
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β_k for $k \in [1:K]$

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Emission probabilities
b_{ik}

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α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

$o_1\ o_2\ o_3\ o_4\ o_5\ o_6$
$\beta_1\ \beta_3\ \beta_1\ \beta_3\ \beta_3\ \beta_2$

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1	0.4200	0.1008	0.0564	0.0135	0.0033	0
α_2	0.0200	0	0.0010	0	0	0.0006
α_3	0	0.0336	0	0.0045	0.0022	0.0003

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1	1	1	1	1	1
α_2	1	1	1	1	3
α_3	1	3	1	3	3

$i_6 = 2$

Backtracking

$$i_N = \operatorname*{argmax}_{j\in[1:I]} \mathbf{D}(j,n)$$

$$i_n = \mathbf{E}(i_{n+1},n)$$

Viterbi Algorithm: Example

HMM:

States
α_i for $i \in [1:I]$

Observation symbols
β_k for $k \in [1:K]$

State transition probabilities
a_{ij}

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Emission probabilities
b_{ik}

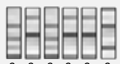
B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

Initial state probabilities
c_i

C	α_1	α_2	α_3
	0.6	0.2	0.2

Input

Observation sequence
O = (o_1,o_2,o_3,o_4,o_5,o_6)



$\beta_1\ \beta_3\ \beta_1\ \beta_3\ \beta_3\ \beta_2$

Viterbi algorithm

D	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$	$o_6=\beta_2$
α_1	0.4200	0.1008	0.0564	0.0135	0.0033	0
α_2	0.0200	0	0.0010	0	0	0.0006
α_3	0	0.0336	0	0.0045	0.0022	0.0003

E	$o_1=\beta_1$	$o_2=\beta_3$	$o_3=\beta_1$	$o_4=\beta_3$	$o_5=\beta_3$
α_1	1	1	1	1	1
α_2	1	1	1	1	3
α_3	1	3	1	3	3

$i_6 = 2$

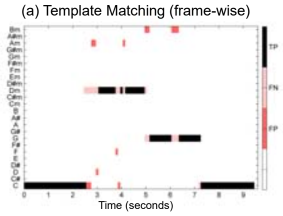
Output

Optimal state sequence
S* = ($\alpha_1,\alpha_1,\alpha_1,\alpha_3,\alpha_3,\alpha_2$)

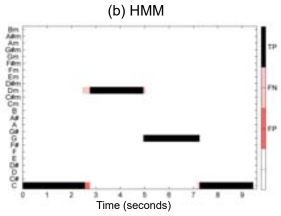
HMM: Application to Chord Recognition

- Effect of HMM-based chord estimation and smoothing:

(a) Template Matching (frame-wise)

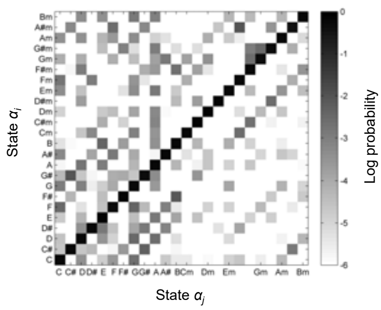


(b) HMM



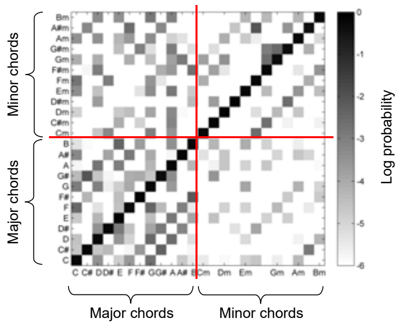
HMM: Application to Chord Recognition

- Parameters: **Transition probabilities**
- Estimated from data



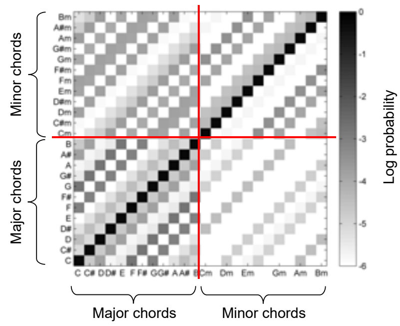
HMM: Application to Chord Recognition

- Parameters: **Transition probabilities**
- Estimated from data



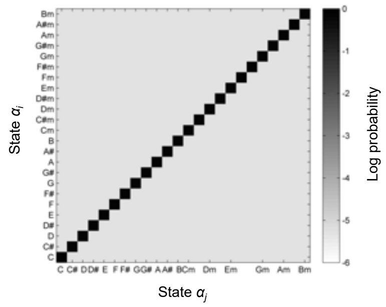
HMM: Application to Chord Recognition

- Parameters: **Transition probabilities**
- Transposition-invariant



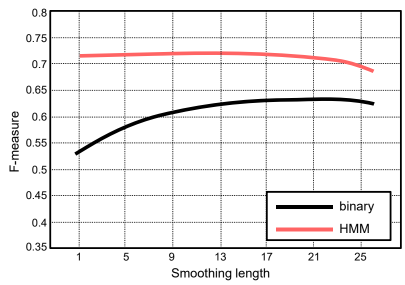
HMM: Application to Chord Recognition

- Parameters: **Transition probabilities**
- Uniform transition matrix (only smoothing)



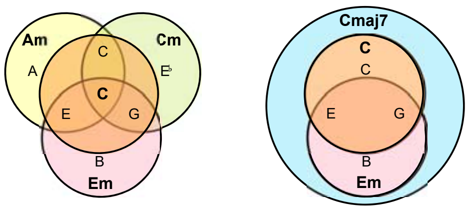
HMM: Application to Chord Recognition

- Evaluation on all Beatles songs



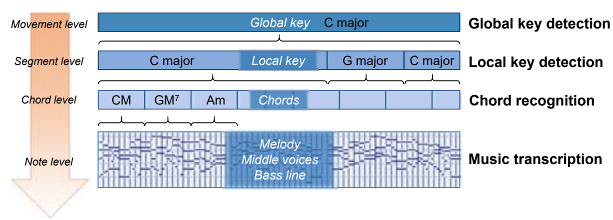
Chord Recognition: Further Challenges

- Chord ambiguities

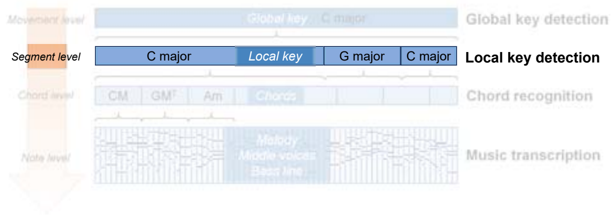


- Acoustic ambiguities (overtones)
  - Use advanced templates (model overtones, learned templates)
  - Enhanced chroma (logarithmic compression, overtone reduction)
- Tuning inconsistency

Tonal Structures

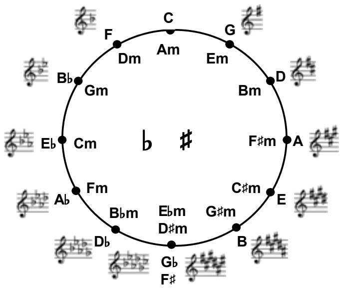


Tonal Structures



Local Key Detection

- Key as an important musical concept (*"Symphony in C major"*)
- Modulations → Local approach
- Key relations: Circle of fifth (**keys**)



Local Key Detection

- Key as an important musical concept (*"Symphony in C major"*)
- Modulations → Local approach
- Diatonic Scales
  - Simplification of keys
  - Perfect-fifth relation

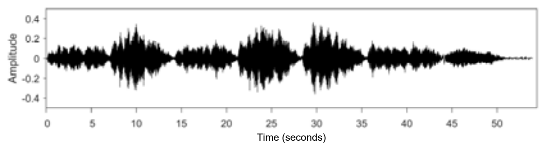
Circle of fifths (**pitches**) →

Local Key Detection

- Example: J.S. Bach, Choral "Durch Dein Gefängnis" (*Johannespassion*)
- **Score** – Piano reduction

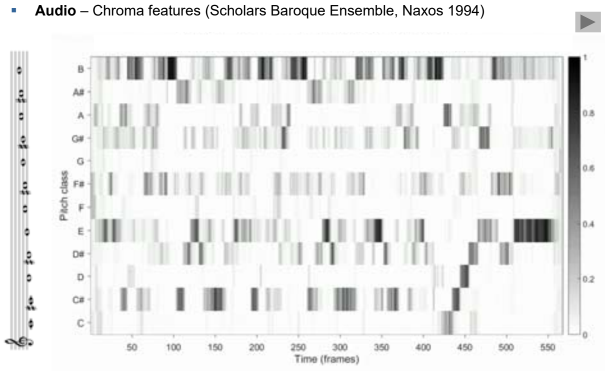
Local Key Detection

- Example: J.S. Bach, Choral "Durch Dein Gefängnis" (*Johannespassion*)
- **Audio** – Waveform (Scholars Baroque Ensemble, Naxos 1994)



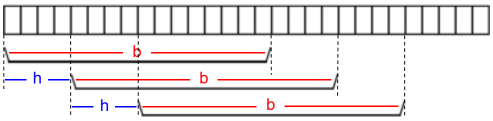
Local Key Detection: Chroma Features

- Example: J.S. Bach, Choral "Durch Dein Gefängnis" (*Johannespassion*)
- **Audio** – Chroma features (Scholars Baroque Ensemble, Naxos 1994)



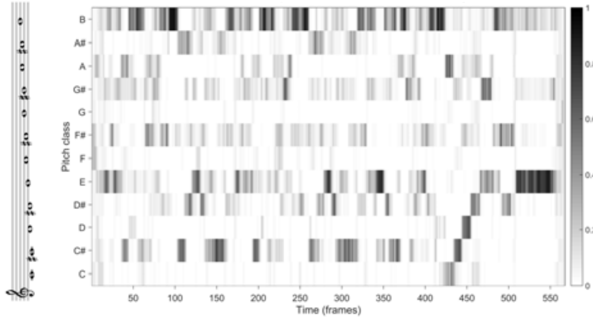
Local Key Detection: Chroma Smoothing

- Summarize pitch classes over a certain time
  - **Chroma smoothing** (mean filter)
  - Parameters: blocksize b and hopsize h



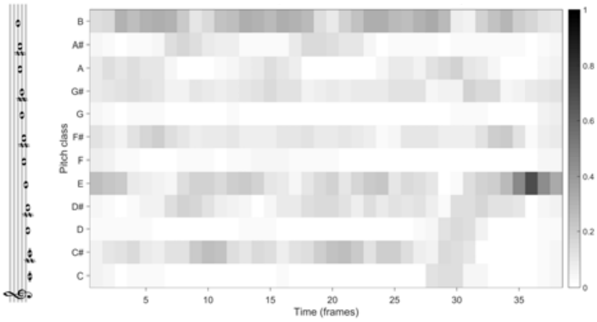
Local Key Detection: Chroma Smoothing

- Choral (Bach)



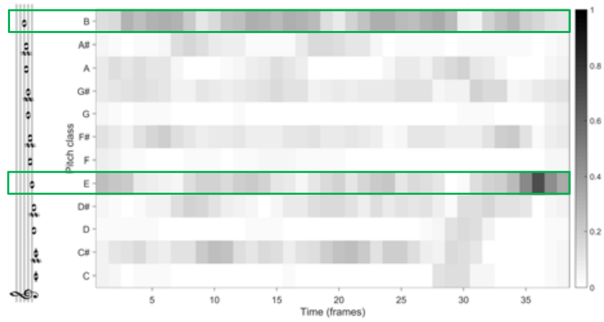
Local Key Detection: Chroma Smoothing

- Choral (Bach) — smoothed with b = 4.2 seconds and h = 1.5 seconds



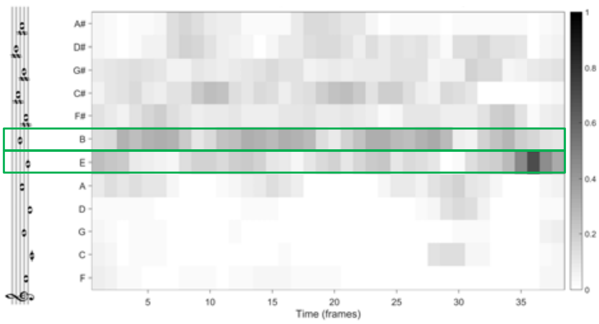
Local Key Detection: Diatonic Scales

- Choral (Bach) — Re-ordering to **perfect fifth** series



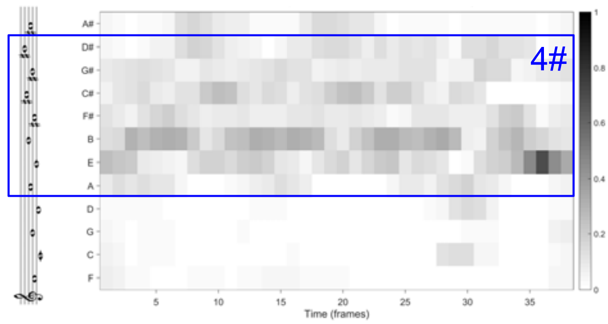
Local Key Detection: Diatonic Scales

- Choral (Bach) — Re-ordering to **perfect fifth** series



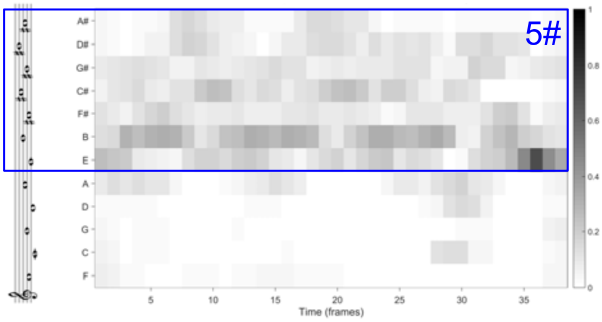
Local Key Detection: Diatonic Scales

- Choral (Bach) — Diatonic Scale Estimation (7 fifths)



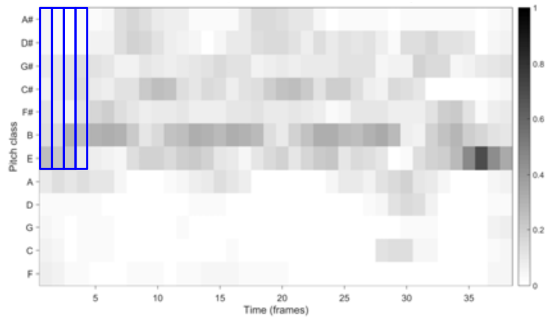
Local Key Detection: Diatonic Scales

- Choral (Bach) — Diatonic Scale Estimation (7 fifths)



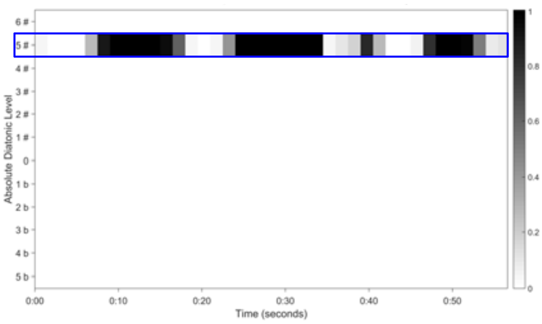
Local Key Detection: Diatonic Scales

- Choral (Bach) — Diatonic Scale Estimation: Multiply chroma values*



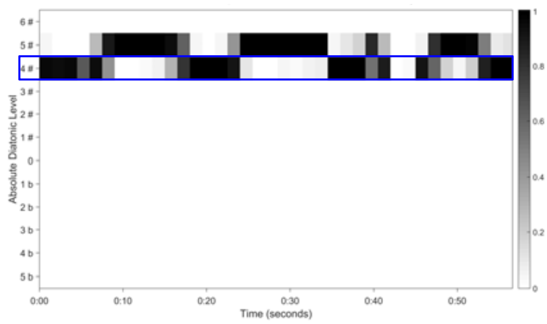
Local Key Detection: Diatonic Scales

- Choral (Bach) — Diatonic Scale Estimation: Multiply chroma values



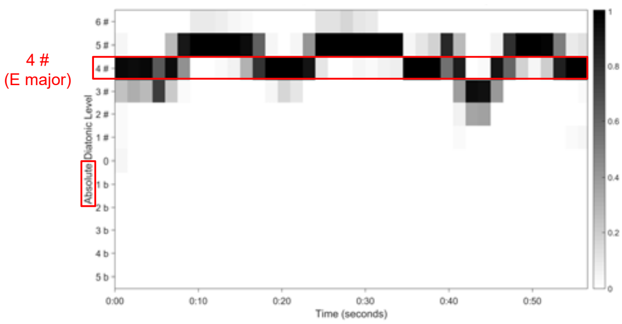
Local Key Detection: Diatonic Scales

- Choral (Bach) — Diatonic Scale Estimation



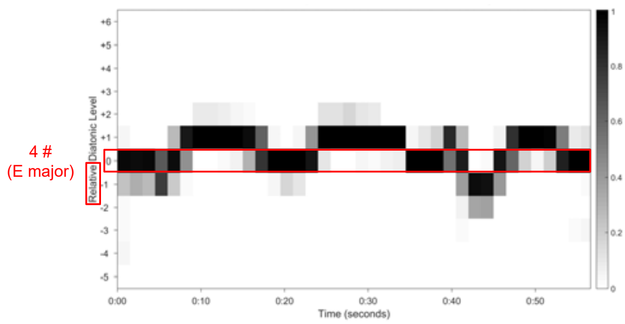
Local Key Detection: Diatonic Scales

- Choral (Bach) — Diatonic Scale Estimation



Local Key Detection: Diatonic Scales

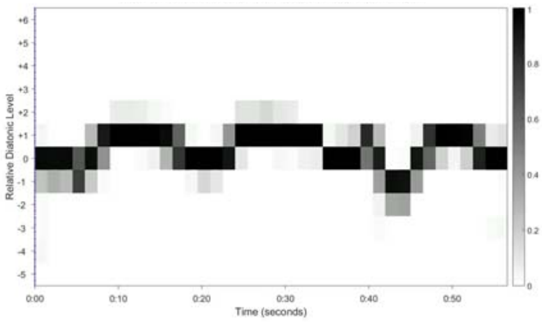
- Choral (Bach) — Diatonic Scale Estimation: Shift to **global key**



Local Key Detection: Diatonic Scales

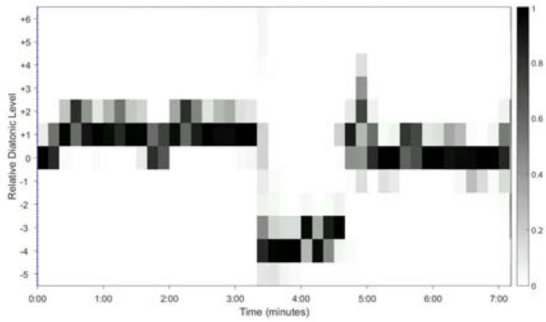
- Choral (Bach) — 0 ≙ 4#

Weiss / Habryka, *Chroma-Based Scale Matching for Audio Tonality Analysis*, CIM 2014



Local Key Detection: Examples

- L. v. Beethoven – Sonata No. 10 op. 14 Nr. 2, 1. Allegro — 0 ≙ 1 (Barenboim, EMI 1998)



Local Key Detection: Examples

- R. Wagner, *Die Meistersinger von Nürnberg*, Vorspiel — 0 ≙ 0 (Polish National Radio Symphony Orchestra, J. Wildner, Naxos 1993)