Music Processing Analysis

Fourier Analysis II

Exercise

Stefan Balke, Meinard Müller

FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

Fraunhofer IIS

Session Outline
Music Representations

- Homework discussion
- Frequency Analysis in Python

Homework

Exercise 2.3

Exercise 2.3. Based on (2.27) and (2.28), compute the time resolution (in ms) and frequency resolution (in Hz) of a discrete STFT based on the following parameter settings:

(a) $F_s = 22050, N = 1024, H = 512$
(b) $F_s = 48000, N = 1024, H = 256$
(c) $F_s = 4000, N = 4096, H = 1024$

What are the respective Nyquist frequencies?

Homework
Solution 2.3

Solution to Exercise 2.3. The time resolution (in ms) is given by $1000 \cdot H/F_\mathrm{s}$, the frequency resolution (in Hz) by F_s/N, and the Nyquist frequency by $F_\mathrm{s}/2$. From this one obtains:

(a) Time resolution: 23.22 ms. Frequency resolution: 21.53 Hz. Nyquist frequency: 11025 Hz.

(b) Time resolution: 5.33 ms. Frequency resolution: 46.88 Hz. Nyquist frequency: 24000 Hz.

(c) Time resolution: 256.00 ms. Frequency resolution: 0.98 Hz. Nyquist frequency: 2000 Hz.

Homework

Exercise 2.4

Exercise 2.4. Let $F_s = 44100$, $N = 2048$, and $H = 1024$ be the parameter settings of a discrete STFT $\mathcal{X}$ as defined in (2.26). What is the physical meaning of the Fourier coefficients $\mathcal{X}(1000,1000)$, $\mathcal{X}(17,0)$, and $\mathcal{X}(56,1024)$, respectively? Why is the coefficient $\mathcal{X}(56,1024)$ problematic?

Homework

Solution 2.4

Solution to Exercise 2.4. According to (2.27) and (2.28), the coefficient $\mathcal{X}(1000, 1000)$ corresponds to the physical time $T_{\mathrm{coef}}(m) = 23.22$ sec and the physical frequency $F_{\mathrm{coef}}(1000) = 21533$ Hz. Similarly, one obtains $T_{\mathrm{coef}}(17) = 0.39$ sec and $F_{\mathrm{coef}}(0) = 0$ Hz for $\mathcal{X}(17, 0)$. Furthermore, one obtains $T_{\mathrm{coef}}(56) = 1.30$ sec and $F_{\mathrm{coef}}(1024) = 22050$ Hz for $\mathcal{X}(56, 1024)$. The frequency expressed by the coefficient $\mathcal{X}(56, 1024)$ corresponds to the Nyquist frequency. In general, this coefficient yields a poor approximation of the actual frequency of the underlying analog signal.

Homework

Exercise 2.15

Exercise 2.15. Let $\exp(i\gamma) := \cos(\gamma) + i\sin(\gamma)$, $\gamma \in \mathbb{R}$, be the complex exponential function as defined in (2.67). Prove the following properties (see (2.68) to (2.71)):

(a) $\exp(i\gamma) = \exp(i(\gamma + 2\pi))$

(b) $|\exp(i\gamma)| = 1$

(c) $\overline{\exp(i\gamma)} = \exp(-i\gamma)$

(d) $\exp(i(\gamma_1 + \gamma_2)) = \exp(i\gamma_1)\exp(i\gamma_2)$

(e) $\dfrac{d\exp(i\gamma)}{d\gamma} = i\exp(i\gamma)$

[**Hint:** To prove (d), you need the trigonometric identities $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ and $\sin(\alpha + \beta) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$. In (e), note that the real (imaginary) part of a derivative of a complex-valued function is obtained by computing the derivative of the real (imaginary) part of the function.]

Homework
Solution 2.15a

$$\textbf{(a)}\quad \exp(i\gamma) = \exp(i(\gamma + 2\pi))$$

Property (a) follows from

$$\exp(i\gamma) = \cos(\gamma) + i\sin(\gamma) = \underbrace{\cos(\gamma + 2\pi) + i\sin(\gamma + 2\pi)}_{2\pi \text{ periodic}} = \exp(i(\gamma + 2\pi)).$$

Homework

Solution 2.15b

(b) $|\exp(i\gamma)| = 1$

Using $\cos(\alpha)^2 + \sin(\alpha)^2 = 1$, property (b) follows from

$$|\exp(i\gamma)| = \sqrt{\cos(\gamma)^2 + \sin(\gamma)^2} = 1\,.$$

Homework

Solution 2.15c

$$\textbf{(c)} \quad \overline{\exp(i\gamma)} = \exp(-i\gamma)$$

Using $\cos(\alpha) = \cos(-\alpha)$ and $\sin(\alpha) = -\sin(-\alpha)$, property (c) follows from

$$\overline{\exp(i\gamma)} = \cos(\gamma) - i\sin(\gamma) = \cos(-\gamma) + i\sin(-\gamma) = \exp(-i\gamma).$$

Homework
Solution 2.15d

$$\textbf{(d)}\quad \exp(i(\gamma_1+\gamma_2)) = \exp(i\gamma_1)\exp(i\gamma_2)$$

Using the two trigonemetric identities specified in the hint, property (d) follows from

$$\begin{aligned}
\exp(i(\gamma_1+\gamma_2)) &= \exp(i\gamma_1)\exp(i\gamma_2) \\
&= \cos(\gamma_1+\gamma_2)+i\sin(\gamma_1+\gamma_2) \\
&= \cos(\gamma_1)\cos(\gamma_2)-\sin(\gamma_1)\sin(\gamma_2)+i(\cos(\gamma_1)\sin(\gamma_2)+\sin(\gamma_1)\cos(\gamma_2)) \\
&= (\cos(\gamma_1)+i\sin(\gamma_1))\cdot(\cos(\gamma_2)+i\sin(\gamma_2)) \\
&= \exp(i\gamma_1)\exp(i\gamma_2).
\end{aligned}$$

[**Hint:** To prove (d), you need the trigonometric identities $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ and $\sin(\alpha+\beta) = \cos(\alpha)\sin(\beta)+\sin(\alpha)\cos(\beta)$. In (e), note that the real (imaginary) part of a derivative of a complex-valued function is obtained by computing the derivative of the real (imaginary) part of the function.]

Homework
Solution 2.15e

$$\textbf{(e)} \quad \frac{d\exp(i\gamma)}{d\gamma} = i\exp(i\gamma)$$

The property (e) follows from

$$\frac{d\exp(i\gamma)}{d\gamma} = \frac{d\cos(\gamma)}{d\gamma} + i\frac{d\sin(\gamma)}{d\gamma} = -\sin(\gamma) + i\cos(\gamma)$$
$$= i(\cos(\gamma) + i\sin(\gamma)) = i\exp(i\gamma).$$